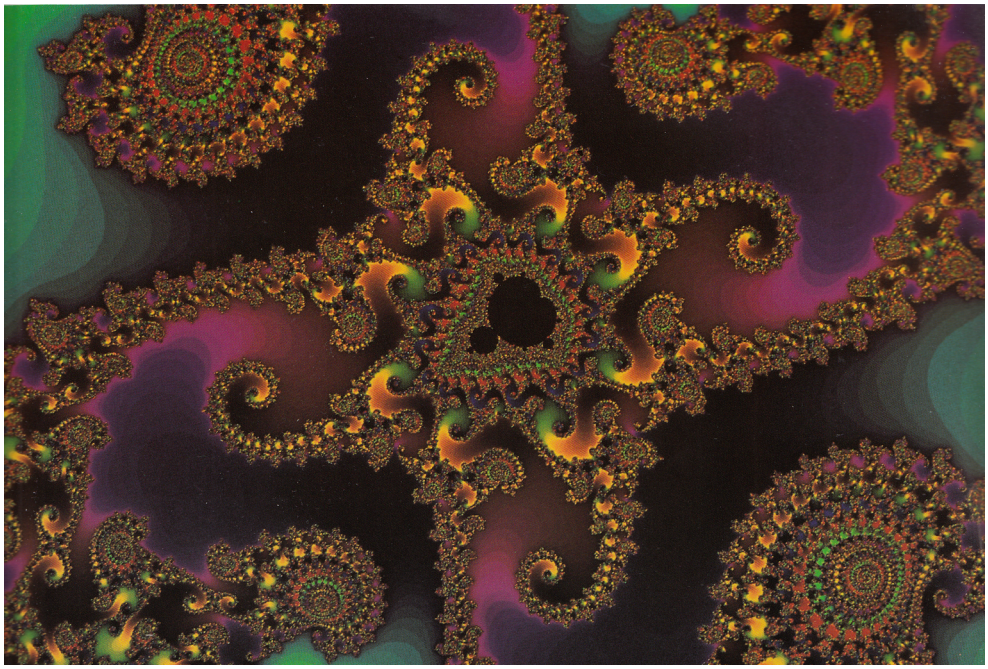
Second Draft

Through Evolution's Accumulation Point

Towards Its Glorious Culmination



Paul Hague

Through Evolution's Accumulation Point

Towards Its Glorious Culmination

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Paragonian Publications
Svenshögen, Sweden

Published by:

Paragonian Publications
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Sweden

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The most up-to-date, searchable version of this book is available at:
http://mysticalpragmatics.net/documents/through_evolutions_accumulation_point.pdf

First edition, 2016

ISBN: 978-91-975176-7-6, 91-975176-7-4

Typeset in Adobe Caslon 12/15

The fractal on the front cover is a jewel box version of the detail in a Mandelbrot set, computed by Rollo Silver of Amygdala in San Christabal, New Mexico.

For
all children everywhere, born and yet to be born, for you are destined
to be carried to evolution's Glorious Culmination in
the much longed-for Age of Light

Contents

About the author

Paul Hague was born near London in the middle of the Second World War, early disquieting experiences that led to a lifelong search for Love and Peace, Wholeness and the Truth, and Life and Freedom. After being educated mainly as a mathematician, he then spent his business career in the information technology industry, primarily with IBM in sales and marketing in London in the 1960s and 70s and in software development in Stockholm in the 1990s, when he took early retirement.

In 1980, as the Information Society was being born, he saw that the global economy holds the seeds of its own destruction within it and that his children were not being educated to live in the world that would exist when they came to have children of their own. Most significantly, he realized that the computer—as a tool of thought—cannot be understood within the context of materialistic, mechanistic science and monetary economics.

Being concerned about our ignorance of what is causing scientists and technologists, like himself, to drive the pace of evolutionary change at exponential rates of acceleration, he resigned from IBM to investigate the long-term psychological and economic implications of society's growing dependency on information technology.

The trigger for this radical change of direction in his life was a life-changing epiphany on 27th April 1980, when, in an apocalyptic eureka moment, he realized that nonphysical, mental, synergistic energies are driving the pace of change in society at unprecedented rates of acceleration. Accordingly, he set out to develop a coherent, self-inclusive map of the Cosmos that would unify the psychospiritual energies acting within us with the four physical forces recognized by materialistic science.

Specifically, to realize the abundant potential of human intelligence, beyond the constraints of artificial general intelligence, Paul imagined that he was a computer that had the task of integrating all knowledge in all cultures and disciplines into a coherent whole without an external human designer to tell it how to do this. Working in solitude, with only the Divine power of Life and Heraclitus' Hidden Harmony for guidance, this computer had the assignment to program itself to develop the Theory of Everything, a coherent body of knowledge that can explain all our experiences, from the mystical to the mundane.

As a consequence of Paul's awakening thought experiment, he has realized that opposites do not exist in Nonduality and Wholeness, fulfilling a childhood dream to end the long-running wars between science and religion and between all the religions, necessary if we are to live in love, peace, and harmony with each other and our environment.

Introduction

This essay, later to be published in extended form as a book, is complementary to *The Four Spheres: Healing the Split between Mysticism and Science*, which summarizes some thirty-six years of research into the psychodynamics of society in order to answer the most critical unanswered question in science, "What is causing scientists and technologists to drive the pace of scientific discovery and technological invention at unprecedented exponential rates of acceleration?"

What these studies into the root causes of change in society have shown is that evolution is currently passing through the most momentous turning point in its fourteen billion-year history since the most recent big bang. By viewing the entire history of evolution as a single dynamical system, this book uses nonlinear mathematics to show that evolution is currently accelerating through its Accumulation Point, leading us into its Glorious Culmination at the end of time in the Eternal Now.

This unparalleled event directly affects all humans living on our beautiful planet Earth, not the least the millennials and their children, who are destined to make greater changes in their lives than all those that have been made during the past five thousand years. Most significantly, by riding the rapids of change, we now have the wonderful opportunity to become free of the conflicts and suffering that have bedevilled human affairs for millennia, learning to live in love, peace, and harmony with each other and our environment.

It is therefore incumbent on the parents and grandparents of the next generations to give them the freedom that they will need to intelligently adapt to the rapidly changing environment they find themselves in. For any species that does not adapt to its changing environment can expect to survive for very long. *Homo sapiens* is no exception to this fundamental law of the Universe. We sink or swim together.

However, at present, we are falling far short of what is urgently required. The novelist and journalist Geraldine Bedell—a director of the Parent Zone—made this crystal clear on 27th February 2016, when she wrote an article in the *Independent* under the rubric, 'Teenage mental-health crisis: Rates of depression have soared in past 25 years'. She then wrote, "For parents and teachers this is a difficult thing to confront: an epidemic of young people at odds with the world around them is hardly a positive reflection of the society we've created for them," going on to say, "We are educating young people for a world that is unlikely to exist in 20 years' time and, arguably, not equipping them with the skills they need for the one that will."

For myself, after a very long turbulent journey, I have now discovered how I should have been educated as a teenager in order to live in harmony with the fundamental laws of the Universe and thereby realize my fullest potential as a human being. Evolution has guided me into a world that is so simple and elegant that it far surpasses the beauty of any piece of music I have ever heard, any mathematical puzzle that I have ever encountered, or any landscape I have ever been in. The only comparable experience is the Stillness enjoyed in Divine lovemaking with one's beloved.

I call this world *Wholeness*, from Old High German *heil* 'whole', which is cognate with *heilida* 'health' and *heilag* 'holy'. Wholeness is our True Nature, a sacred space where all of us live at every moment of our lives. No one can return Home to Wholeness, for nobody has ever left Home.

So why don't we teach our children to live in Wholeness today? Well, the challenges we face as a civilization and species can be traced to the day, between twenty-five and forty thousand years ago, when our forebears were given the great gift of Self-reflective Intelligence, the Divine quality that distinguishes humans from the other animals and machines, like computers.

This was a momentous event in evolutionary history, necessary so that one day intelligent beings could discover what the Universe is and how the Cosmos is designed. That is the extraordinary beauty of the Universe, that it can reveal its nakedness in this exquisite way. However, when our ancestors set out on their journeys of discovery, they were like babies in adult bodies, with little conceptual understanding of the world they were living in, not being aware of the concept of concept, for instance. Most significantly, while they were learning to deal practically with the vicissitudes of life, this knowledge did not help them understand the Power and Presence of the Divine that they could sense within and around them.

That wasn't the only challenge our ancestors faced as humanity set out on its journey towards evolution's Glorious Culmination. Ever since the most recent big bang, evolution had been more divergent that convergent, a bifurcating process that has continued in the noosphere, terminating today in evolution's Accumulation Point. As a result, our minds have become fragmented, and society, as a collective projection of our minds, has become divided into religious and national factions, academic specialization, and the division of labour in the workplace.

We thus face a perilous situation as both a civilization and a species. Our education and economic systems are designed to protect the status quo, inhibiting us from peacefully and intelligently adapting to our rapidly changing environment. Indeed, our entire cultural infrastructure is preventing humankind from realizing our fullest potential as a superintelligent, superconscious species before our inevitable demise. The root cause of this life-and-death problem is existential fear, which arises when we feel experientially and cognitively separate from the Immortal Ground of Being that we all share.

Such existential fears are being exacerbated today by scientists who claim that they are about to build computers with artificial general intelligence, replacing a substantial proportion of the jobs currently being performed by humans in the workplace. They call this momentous event in evolutionary history a technological singularity in time, which corresponds to the accumulation point in chaos theory. Inspired by R. Buckminster Fuller's *Utopia or Oblivion: The Prospects for Humanity*, they seem to believe that technology can resolve human woes, not intelligent humans, guided by the creative power of Life, bubbling up through us from the Divine Origin of the Universe, like a fountain. Indeed, many deny the very existence of the energies that are needed for us to understand what causes us to behave in the way that we do; for evolution to become fully conscious of itself within us.

Clearly, if we are to adapt to our rapidly changing environment, we need to look afresh at our lives together, free as much as possible from our mechanistic cultural conditioning, which causes us to behave more like human automata than the liberated beings we have the potential to be. The conventional mathematics in the main body of this essay can help us to understand what is happening to humanity at the present time, liberating us from what William Blake aptly called our 'mind-forged manacles'.

But first, if the mathematical symbols are to be interpreted in a meaningful manner, we need to look at the broad picture, exploring the personal, social, and Cosmic aspects of our lives as a coherent whole. For while we are all unique beings with our own particular propensities, there is much that we share beneath the surface, which is revealed through open and honest self-inquiry, lying outside superficial science and business, as they are practiced today.

For myself, I realized that there is something seriously amiss with our education and economic systems in the late 1970s, when working in marketing for IBM in London. With the accelerating pace of change that I was helping to create as an information systems architect in business, I could see that my children, then aged nine and six, were not being educated to live in the world that they would be living in when they came to be bringing up children of their own.

Furthermore, I learned little at school and university in religion, science, economics, and mathematics in the 1950s and 60s because I had realized at seven years of age in 1949 that the culture I had been born into did not have an overall context within which to coherently interpret what I was being taught. Specifically, the concepts of God and universe in Christianity and physics are incompatible with each other. So, until they are reconciled, we shall not be able to end the long-running war between science and religion, essential if we were ever to bring about World Peace.

Accordingly, since 1980 I have been living as an autodidact, seeking to develop a coherent worldview and unifying language that everybody in the world could share, no matter what religion, nation, or culture they had been born into or what speciality they studied in academia or practiced in the workplace.

What I have discovered through self-inquiry—like mystics through the ages—is that we can only live in love, peace, and harmony with our fellow human beings when we are at Peace within ourselves, living consciously in egoless union with the Divine. So, in this book, I would like to introduce this lifelong experiment in learning by first exploring how the ultimate purpose of life on Earth can be realized through nonaxiomatic mathematical reasoning, as a complement to traditional spiritual practices, such as meditation and yoga.

As Richard Courant and Herbert Robbins tell us in their classic textbook *What Is Mathematics?*, at the heart of mathematical logic lies a pair of complementary ideas that originated in Aristotle's *Metaphysics*. They are the Laws of Contradiction and Excluded Middle. The former states, "An entity cannot possess both attributes A and not-A," not-A variously written as A', $\sim A$, and $\neg A$. Its complement states, "An entity must either possess a given attribute or not possess it."

In mathematics, A and A' are interpreted as either the values 'true' and 'false' or elements in and out of a set, respectively, which can be depicted in Euler-Venn diagrams, taught to children in schools today. These are then combined in various ways, corresponding to addition and multiplication in arithmetic, as George Boole showed in laying down the foundations of mathematical logic in 1853 with his magnum opus *An Investigation of the Laws of Thought on Which Are Founded the Mathematical Theories of Logic and Probabilities*. This book was a development of a prize-winning paper on general operator theory that Boole wrote in 1844, following a mystical experience he had had as a seventeen-year-old, twelve years earlier. Since then, mathematics has been the science of patterns and relationships, more general than the science of space and number, as the Ancient Greeks understood mathematics.

Furthermore, *Laws of Thought* began the process of unifying the split between mathematical logic and depth psychology—as sciences of the mind—which Charles Sanders Peirce, Gottfried Frege, and Bertrand Russell were later to open up again. For this book set out to show how we can use mathematics to understand "the nature and constitution of the human mind", understanding "the fundamental laws of those operations of the mind by which reasoning is performed".

Laws of Thought led to the invention of the stored-program computer a century later. However, this book and its successors don't really tell us how humans think and reason, for the computer is essentially a linear device, executing instructions sequentially, albeit in many parallel threads in modern multi-headed central-processing units, collectively collaborating in networks, such as the Internet. In contrast, we

humans think nonlinearly, cross-referencing the multitudes of relationships that transcend all disciplines and cultures, as we see in the hyperlinks on the Internet, which we are constantly clicking. So axiomatic deductive reasoning, which is used to prove the truth of mathematical theorems, cannot lead us to the Truth. Furthermore, as the systems philosopher Ervin Laszlo has said, we need to give up the idea that the world is a giant mechanism. Rather the Universe is "most like an Internet, a kind of Cosmic Internet."

Rather than using the arithmetic operators + and × to denote the logical relationships between entities, in modern mathematical language, + becomes logical-or or disjunction ∨ and union ∪, and × becomes logical-and or conjunction ∧ and intersection ∩. The Laws of Contradiction and Excluded Middle are then mathematically expressed in this table:

	Logic	**Sets**
Contradiction	$A \wedge A' = 0$	$A \cap A' = 0$
Excluded Middle	$A \vee A' = 1$	$A \cup A' = 1$

These equations are examples of the principle of duality in Boolean algebra, in which the symbols ∧ and ∨, ∩ and ∪, and 0 and 1 in any theorem can be changed into their duals giving another valid theorem. The principle of duality also appears in projective and inversive geometry, where lines and points are duals of each other, giving dual figures and theorems about them.

We see here, at the heart of mathematics, the universal principle that whenever we form a concept we always form its polar or dual opposite. For myself, I first saw how the mathematical principle of duality could be generalized to apply to all beings at midsummer 1980, when I formulated the Principle of Duality: *A complete conceptual model of the Universe consists entirely of dual sets*.

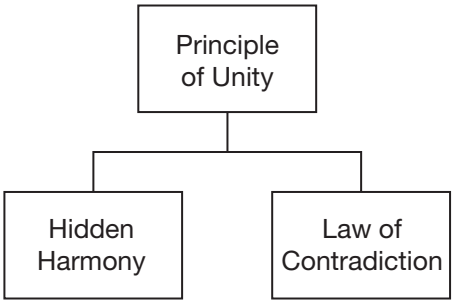
When this idea was revealed to me in an explosive flood of creative energy, I began to wonder where it had come from. For, I realized at once that I had been given the key that would unlock the innermost secrets of the Universe that had puzzled me since I was a boy. Knowing nothing about mystics and depth psychologists, I began to look deeply into myself, a process of self-discovery I had begun in 1974 on IBM management education courses. This enabled me to form the concept of the Absolute in October 1983 in exactly the same way as I form any other concept. The Principle of Duality then became the all-embracing Principle of Unity, expressed in just seven words: *Wholeness is the union of all opposites* or six mathematical symbols, which I also call the Cosmic Equation, where A is any being, W is any whole, including Nondual Wholeness, ∪ is union, and ~ is not:

$$W = A = A \cup {\sim}A$$

The Principle of Unity, which is an irrefutable, universal truth, has no predecessors in the entire history of human learning because it emerges directly from the Divine Origin of the Universe. As the Cosmic Equation, it is a mathematical theorem that cannot be proven to be true from any set of axioms, which are characteristics of linear systems of thought, not nonlinear. So if we are to give mathematics and hence the whole of human learning a solid foundation, we need to abandon Euclid's notion of self-evident or assumed postulates, which began the time-honoured *The Elements*, and to let go of all other axiomatic systems developed since then.

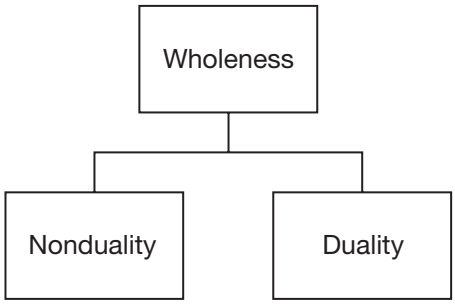
Aristotle, himself, expressed the Law of Contradiction with these words, "It is impossible for the same attribute at once to belong and not to belong to the same thing and in the same relation … as some imagine Heraclitus says." For, as the mystical philosopher of change Heraclitus of Ephesus well knew, opposites, even contradictory ones, can never be separated, a fundamental principle that he called the Hidden Harmony: "Opposition brings concord; out of discord comes the fairest harmony."

So, following Heraclitus, we can use the limitless explanatory and creative power of the Principle of Unity to end the argument between Heraclitus and Aristotle, as this diagram illustrates. There is a primary-secondary relationship between the Principle of Unity and the Law of Contradiction, one of many examples of such primary-secondary relationships.



However, while I was able to use the Principle of Unity to consistently form the concept of the Absolute in 1983, it was to take twenty years attending many psychospiritual workshops and retreats before I had irrefutable empirical evidence for the Truth, as I went through a series of satoris or kenshos in the terms of Zen Buddhism in the mountains of Norway and forests of Sweden. God had become a scientific concept, gnostically understood with Absolute Certainty.

Most significantly, I could then view the Totality of Existence as both the Formless Absolute and the relativistic world of form, in which these two beings are never separate from each other, with the former being the meaningless and primal Supreme Being, from which all forms, structures, and relationships emerge in a meaningful manner. This emergence is the first bifurcation in the Universe, which can be reunited when we realize that Wholeness is the union of Nonduality and Duality, illustrated here in another instance of the Principle of Unity. Like Hegelian logic, if *A* and not-*A* are thesis and antithesis, respectively, then *A* is the synthesis.



However, there is a very great danger here, as Carl Gustav Jung pointed out in *Aion* and other works. Historically, there has been a tendency for the Absolute to split into God—as the Christ—and Satan or the Devil. What is regarded as evil rather than good then becomes suppressed in the unconscious psyche as the shadow, of which there are many archetypes. What people call their demons in popular culture then become projected on to others, leading to much conflict and suffering.

This Western way of separating pairs of opposites is quite different from that in the East. In Jung's *Commentary* to Richard Wilhelm's translation of *The Secret of the Golden Flower*, he said, "The Chinese have never failed to recognize the paradoxes and the polarity inherent in all life. The opposites always balance on the scales—a sign of high culture. Onesideness, though it lends momentum, is a mark of barbarism." For instance, American presidents, as commanders-in-chief of the US military, often end their speeches with the words, "God bless America."

But why not bless everyone on Earth? Doesn't everybody deserve the best that Life can give us? We are all human beings, faced with essentially the same challenges in life. We are all born to die, an inevitability that has deeply troubled humanity for tens of thousands of years, as the only species that is aware of this fundamental law of existence. We can resolve this central issue of our lives through meditation practices. As Shakyamuni Buddha said on his deathbed, "Behold, O monks, this is my last advice to you. All component things in the world are perishable. They are not lasting. Strive on with diligence."

In mathematics, there are many instances of the Cosmic Equation, not only in the principle of duality. Examples of complementary opposites include continuous and discrete functions, convergent and divergent series, exponents and logarithms, and differentiation and integration, known as the fundamental theorem of the infinitesimal calculus. A simple example of a self-contradiction is the statement, "This sentence is false," which is both true and false. Another example of a paradox in mathematics is the supposition that the infinite set of all sets is the largest possible, for Georg Cantor

showed that the set of all subsets of any set, even infinite ones, is larger than the set itself, leading to a major crisis in the foundations of mathematics.

This crisis cannot be resolved until we realize that Wholeness is the Ultimate Being, which embraces all opposites in Nonduality, whether they be complementary or contradictory. The Cosmic Equation thus gives mathematics a solid foundation, just as the Principle of Unity does the same for all knowledge. So when we embrace the Hidden Harmony in the depth and breath of consciousness, we can live in harmony with the fundamental law of the Universe, extricating Western thought from the evolutionary cul-de-sac it finds itself in today. Nothing is more important.

To give you some more background on how I came to discover the Principle of Unity lying at the heart of the Cosmos, I need to go back a few years to 1977, when Life drove me into an abyss and has since showed me the way out. At the time I was working as a first-line manager in an IBM sales office in London.

Seeking to extricate myself from my midlife crisis, which brought my managerial business career to an abrupt end, I began to develop a national marketing programme for Decision Support Systems, in which managers and professionals, such as scientists, engineers, and accountants, were doing their own personal computing attached interactively to mainframes and mini computers. When giving presentations on this major watershed in the history of the data-processing industry at customer seminars at IBM's European Education Centre in Belgium, information systems directors asked me what these changes would mean for skills profiles, employment, and the quality of life in the workplace in the long term, which I interpreted as thirty years rather than the five that the questioners probably implied.

I did not know the answer to these questions, which require far greater self-understanding than I had at the time. As I have since discovered, the key to such self-understanding is to study the essential differences between human and so-called artificial intelligence. As described in *The Four Spheres*, ever since 1956, computer scientists have been predicting that machines capable of replacing a significant proportion of human jobs will enter the workplace within ten to twenty-five years. I did not pay much attention to these prognostications during the first dozen years of my business career.

However, in the winter of 1979/80, I began to explore whether it would ever be possible for a computer to do my job as an information systems architect. To explain this, in order to obey the economic imperative of our times—replacing as many jobs performed by humans by machines as possible—information systems architects develop models of dynamic business processes, such as designing, manufacturing, marketing, ordering, and invoicing, and their relationships to each other, as well as integrated models of static classes of information in enterprises, such as employees, customers, products, locations, and deliveries.

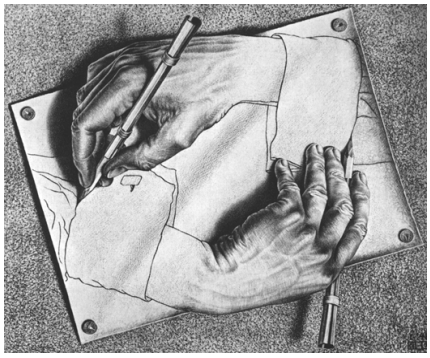
But what about the tasks of staff doing their own personal computing and the jobs of software developers? Could these also be automated? I had begun to be interested in interactive computing in 1974, when I was the systems-engineering manager responsible for ensuring that the first computer that the British Post Office (now British Telecom) bought from IBM met its design specifications. At the time, scientists, technologists, and financiers communicated mainly through typewriter terminals. In IBM itself, APL (A Programming Language) was used as the principal management information tool.

Forty years later, things have moved on considerably, but the underlying concepts are still the same. For instance, they are contained within the amazing Jupyter Notebook running under a Web browser, supporting scientists and statisticians doing data analysis and data mining, programming in a variety of

languages, such as Python, Julia, and R, even able to operate with symbolic mathematics, like Mathematica, the first product with such extraordinary capabilities.

As open-source software, the Jupyter Notebook is so innovative and becoming so popular for scientific computing that the scientific journal *Nature* published an article on this elegant and powerful tool in November 2014, with an illustrative, interactive notebook on its website, of course. In researching this essay, I have used the IPython and Matplotlib components in Jupyter a little to explore how fractals and chaotic phenomena can be presented graphically.

But back in 1980, there was one job that interested me particularly. Could a superintelligent machine replace the job of information systems architect? At the time, the computer tools for doing such semantic and process modelling were extremely primitive, if they existed at all. Since then, I have had a little experience of modern modelling tools from working as a computer consultant on the class and data models for a company developing innovative software for investment banking in Stockholm's World Trade Center in the early years of this century.



But could a computer program itself to develop comprehensive models of business processes without human intervention? If so, this computer would need to include its own creative thought processes in the territory being mapped, which looks rather tricky, for it would be like a television camera filming itself filming, brilliantly illustrated by Escher's famous lithograph 'Drawing Hands'. As the satirical news outlet *The Onion* observed in 2014, you cannot use the mind to study the mind. So what is the point of psychology, as the science of the mind?

The beginnings of a solution to this apparently intractable problem was given to me at 11:30 on Sunday, 27th April 1980, when what I now describe as a big bang erupted at the Divine Origin of the Cosmos, in the depths of the psyche. As I interpret this experience today, this was evolution's way of telling me to start my life afresh, free from the past, rebuilding the entire world of learning at the very beginning so that all the divergent streams of evolution would one day converge at its Glorious Culmination.

Specifically, what triggered this apocalyptic happening was the realization that psychospiritual energies are causing scientists and technologists, like myself, to drive the pace of evolutionary change at exponential rates of acceleration. I knew this immediately in my own direct experience, for the irrepressible creative energies pouring through me could not be explained in terms of the four forces recognized by the physicists: electromagnetic, gravitational, and weak and strong nucleic forces. So to explain what was happening to me and to society as a whole, I set out to develop a radically new map of the Cosmos, one that would integrate the nonphysical and physical energies at work in the Universe into a coherent whole.

However, before this could happen, I was plunged into an even deeper abyss than I had fallen into in 1977. My entire life disintegrated into primordial chaos. Within just a couple of years, my first wife and I divorced, I abandoned my managerial business career, and I eventually became estranged from my two children for I could not explain to them what was happening to me. Cognitively, the

Tower of Babel, which depicts the confused state of the world of learning today, was razed to the ground, rather like a tsunami in which everything is destroyed when it reaches the shoreline.

Then, eight weeks later, around midsummer 1980, I was given the means by which I could extricate myself from the primordial chaos I found myself in. For *primordial* derives from Latin *prīmordium* 'origin, commencement', from *prīmus* 'first' and *ordīrī* 'to begin to weave'. During these eight weeks, I had been seeking a mathematical tool that would provide me with the framework that I needed to integrate all knowledge into a coherent whole. I began with sets, propositional calculus, projective geometry, and catastrophe theory, which a British colleague in Paris had introduced me to the December before, knowing that I could foresee the catastrophe that the global economy was blindly accelerating towards.

Now these nonnumerical, topological mathematical techniques have one pattern in common. At their cores lie pairs of opposites: in and out, true and false, point and line, and smooth and abrupt change, respectively. It was while puzzling about this common pattern in various branches of mathematics that evolution revealed its innermost secret, which I now call the Principle of Unity or Cosmic Equation.

This was another apocalyptic moment, like an after-shock following a massive earthquake in the depths of the Ocean of Consciousness, generating another tsunami. For *apocalypse* derives from Greek *apokalupsis*, from *apokaluptein* 'to uncover' or 'to reveal', from the prefix *apo* 'from, away' and *kaluptra* 'veil'. So *apocalypse* literally means 'draw the veil away from', indicating the disclosure of something hidden from the mass of humanity—Heraclitus' Hidden Harmony.

But now, nearly eight weeks after my original epiphany, I had been shown the way out of the primordial chaos I found myself in. By starting afresh at the very beginning, I had the means by which to rebuild the entire world of learning, weaving all opposites into the most beautiful web, like Indra's Net in the *Avatamsaka Sutra* (*Flower Ornament Scripture*) in *Huayan Buddhism*.

As I have since discovered, the Principle of Unity is not a new idea for those who have looked deeply inside themselves. For instance, this irrefutable, universal truth lies at the heart of Daoism, as indicated by Laozi in *Tao Te Ching*: "When all the world recognizes beauty as beauty, this in itself is ugliness. When all the world recognizes good as good, this in itself is evil." Similarly, in the Christian Book of Revelation, meaning 'Apocalypse', John of Patmos showed that he understood the ultimate purpose of life on Earth when he said, "I am Alpha and Omega, the beginning and the end, the first and the last."

The Hidden Harmony is thus the Holy Grail in Christianity and alchemy's Philosophers' Stone, the Apotheosis of all human learning. However, Heraclitus and Laozi were well aware how few of their contemporaries had discovered the Universe's best-kept secret, heavily defended by the egoic mind, fearful of venturing too far into the depths of the Cosmic Psyche. For instance, the former said, "People do not understand how that which is at variance with itself agrees with itself." And the latter wrote, "My words are very easy to understand and very easy to practice: But the world cannot understand them nor practice them." Little has changed, even today.

For myself, the Cosmic Equation has helped me to understand why Life destroyed everything in my life, leading me to become a renunciate, sacrificing my family, job, career, and home during the two years from 1979 to 1981, when I was financially destitute but having the greatest adventure of my life. As opposites can never be separated from each other, the Alpha and Omega Points of evolution are indivisible—one and the same. So if evolution is to reach its Glorious Culmination within us, as both individuals and as a species, we have no choice but to start afresh at the very beginning.

Needless to say, those few weeks in the spring of 1980 were the most momentous turning point in my life. For they have led me to understand the Grand Design of the Cosmos and so have enabled me to see

that the business modelling problem I was wrestling with at the beginning of 1980 can only be solved with Self-reflective Intelligence. It is by standing outside ourselves that we can study our own subjective minds objectively. As Meister Eckhart, the pre-eminent Christian mystic, said, "The eye with which I see God is the same as that with which he sees me."

A few months later, I discovered that David Bohm had used Self-reflective Intelligence to solve essentially the same problem: to unify the incompatibilities between relativity and quantum theories. For these two fundamental physical theories display opposite characteristics, the former having the properties of continuity, causality, and locality, with the latter being characterized by noncontinuity, noncausality, and nonlocality. As the Principle of Unity reconciles these incompatibilities in Wholeness, Bohm became my principal scientific mentor in a series of meetings at London University during the 1980s, when my renaissance progressed from conception and birth to infancy and childhood.

Such a quantum leap is not unique, for we have words in our languages to denote such unforeseen changes in direction in our lives, which can be quite awesome, generating both excitement and fear, as we venture into the unknown. For instance, we call such aha experiences epiphanies, which are often spiritual, or sudden insights, which are more cognitive. And of course, Archimedes is said to have cried out "Eureka!" 'I have found it', when he ran naked down the street straight from his bath, when he hit upon a method of determining the purity of gold in an irregular object. In the East, such sudden awakenings are often called *Kundalini*, leading to *jivan-mukti*, from Sanskrit *jīv* 'to live' and *moksha* 'liberation from worldly bonds', being fully alive while still in our bodies, free of existential fear.

To further explain what happened to me after I resigned from IBM, in May 1980 I began a thought experiment, not unlike those that Albert Einstein formulated in developing the special and general theories of relativity. In my case, to determine to what extent computers with so-called artificial intelligence could replace human jobs in the workplace, I imagined that I was a computer that switched itself off and on again so that it had no programs within it, not even a bootstrap program to load the operating system from external storage.

This computer then had to program itself to solve the problem that Einstein spent the last thirty years of his life trying to solve. In other words, starting from a *tabula rasa* 'clean slate', the computer had the task of integrating all knowledge in all cultures and disciplines at all times, past, present, and future, into a coherent whole, leading to what physicists call the theory of everything.

This human approach to investigating human vis-à-vis machine intelligence is in direct contrast to those computer scientists seeking to program computers to beat human beings at their games. For instance, IBM's Deep Blue beat Garry Kasparov in a chess match in May 1997, an IBM computer called 'Watson', named after IBM's founder, beat two champions in the Jeopardy! TV quiz show in February 2011, and Google's DeepMind beat Lee Se-dol at Go, far more complex than chess, in March 2016. In *Superintelligence*, Nick Bostrom gives several other examples of what he calls the 'superhuman' capabilities of machines.

But such capabilities are not evidence of artificial intelligence, as press reports often claim, for they are not based on a deep understanding of what human intelligence really is, in contrast to the intellect. To understand what it truly means to be a human being, Life has led me to invert Alan Turing's imitation game, proposed in 1950 in an article titled 'Computing Machinery and Intelligence', published in the philosophical journal *Mind*. Rather than programming a computer to simulate human reasoning, I have

been engaged in an experiment designed to see if humans could simulate a machine that would solve the ultimate problem of human learning without an external programmer to tell us how to do this.

In brief, what I have been doing for the past thirty-six years is to use the semantic modelling methods that underlie the Internet as a mirror for our own creative thought processes. So that those unfamiliar with the job of information systems architect in business could understand what I am writing about, I have abstracted the essence of the relational model of data and object-oriented modelling methods that underlie the Internet. For this essence is simple commonsense, which anyone can understand.

But now I need to explain something that is outside most people's experience. In order to simulate a machine designed to develop a self-inclusive map of the Cosmos, I have first needed to simulate the bootstrap programs that load the operating system into the machine, using concepts taken from computer science, philosophy, and mathematics. I write the words that signify these bootstrap or primal concepts in **bold** font to distinguish them from descriptive words and concepts.

First, a computer is a machine for processing data, aptly called *dator* in Swedish. So the first bootstrap concept I use is **Datum**, from Latin *dare* 'to give, cause', although these two meanings have different etymologies. In my experience, the Datum of the Universe is the Formless Absolute, the Ultimate Giver and Cause of Life.

Although I don't know of anyone else who has denoted the Absolute with the word *Datum*, such a notion is well known in human experience. Throughout the ages, we humans have sensed an immanent, transcendent Presence, etymologically 'before being' or 'prior to existence', for *Presence* derives from Latin *præsentia* 'presence', participle of *præesse* 'to be before', from *præ* 'before' and *esse* 'to be'. The word *Presence* indicates that the Absolute is the Supreme Cause of Everything there is, which mystics through the ages have sought to reveal.

The entire relativistic world of form thus emerges from the Datum, initially as meaningless **patterns of data** before they are interpreted as meaningful information and knowledge by an intelligent being. To simplify the exposition, I call these data patterns **beings**, the next primal concept, which Aristotle regarded as the most fundamental of all concepts in *Metaphysics*, more elemental and abstract than mathematical concepts.

Interpretation then begins by noticing the **similarities** and **differences** in the **attributes** of these beings, putting them into **sets**, the most basic concept in mathematics, more fundamental than that of number, which is not a bootstrap concept. These sets are then gathered together in **classes** of beings, corresponding to Plato's concepts of universals and particulars in *The Republic*. Beings are **entities** with various **attributes**, corresponding to Aristotle's concepts of subject and predicate in the *Organum*. In summary, we can use the primal concepts of **class**, **entity** (as **instance** of class), and **attribute** to lift ourselves up by our bootstraps.

These basic primal concepts are then used to create the few other bootstrap concepts that are needed to load the complete Integral Operating System that we all use to form concepts and organize our ideas. Foremost among these are the abstract concepts of **structure**, **form**, **relationship**, and **meaning**, far more general than the matter, space, and time of the physicists, and the fire, air, earth, and water of the ancients, which are also not primal concepts.

We can see this most clearly from the fact that mathematicians and computer programmers treat mass, space, and time in exactly the same way as other variables in their functions. For instance, Newton's famous equation $F = ma$ has exactly the same structure as *cost* = *price* × *quantity*, which we use when buying a few kilograms of potatoes in the local supermarket. That is the beauty of algebra, as a Kahn

Academy video published on YouTube in 2012 and watched by over two million people indicates. Nothing could be simpler.

As a rather outrageous example, humanity, as a species, is a being, just like any other. So, in principle, we could apply an operator to this being, just as the operator ⌹ in APL inverts a matrix, requiring several instructions in most other programming languages. But there is no operator that we can apply to *Homo sapiens* to mechanistically effect the contextual inversion that humanity needs to pass through in the next few years. For such a radical transformation of consciousness can only come from within, from energies acting in the vertical dimension of time, not the horizontal.

It is through such an experiment in learning that we can bring universal order and harmony to the cognitive chaos and confusion that politics, economics, business, science, medicine, law, philosophy, mathematics, logic, psychology, and religion are in today.

But before we explore the social aspects of the unprecedented rate of evolutionary change that we are experiencing today, I need to describe another rather unusual characteristic of my journey in life. Life had well prepared me for this experiment in learning because what I had been taught in religion, science, economics, and mathematical logic had made no sense to me as a coherent whole in childhood and adolescence. Unlike my contemporaries, I learned almost nothing at school and university. So I had little to unlearn when I set out on this great adventure at the age of thirty-eight. However, I had been educated sufficiently as a pure mathematician, subsequently developing additional skills I needed for this experiment as a computer scientist and information systems architect in business.

During the first few years of this thought experiment, my learning accelerated at superhyper-exponential rates of acceleration, driven by the creative power of the Logos, emerging directly from the Origin of the Universe. So I had irrefutable evidence from direct experience that there are synergistic nonphysical, psychospiritual energies at work in the Cosmos, as well as the four physical forces accepted by materialistic, mechanistic scientists.

All that remained was to find a way of describing my experiences to those who have yet to pass through evolution's Accumulation Point accelerating towards its Glorious Culmination. It has not been easy, not the least because while I have been able to see the Big Picture since April 1982—when I was helping to develop a management accounting system as a computer consultant in Kuwait—it was very fuzzy at that time, like an old-fashioned chemical photograph at an early stage of development. In the event, it has taken over thirty years exploring the world of learning before the Big Picture became sufficiently clear to communicate it to others.

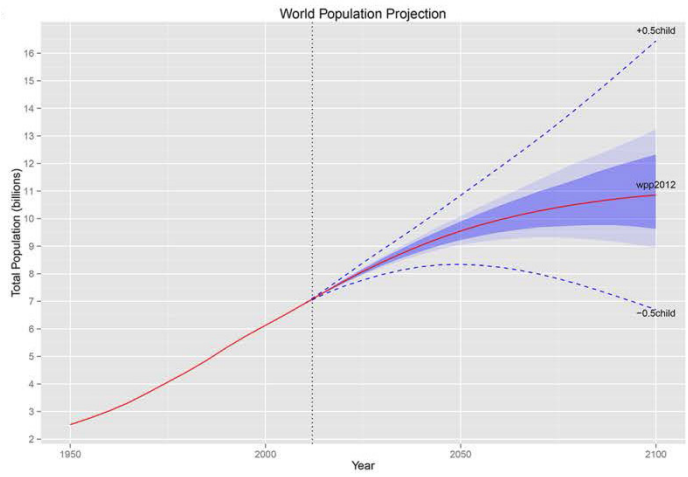
This life experience puts me in a rather unusual situation. I have not painted a picture that can be hung on a wall or invented anything that can be seen by people in my physical environment, such as a new gadget, like a smart phone, or a social-media service on the Internet, used by millions. My physical appearance has changed little over the years other than I look a little older. And although my demeanour changes during the day and from day to day, it is not possible for people to see whence I am coming from my outer appearance and behaviour, for Wholeness is the union of all opposites.

All I can do is express in words, diagrams, and pictures what is within me, rather like a composer writing a symphony. It is then up to my readers to 'hear' the music in the writings so that they too can enjoy the beauty that is thereby revealed. Of course, if we could meet face to face, our mutual learning would accelerate synergistically, as we mirror each other, just two jewels in Indra's net of jewels.

That is enough about my life experiences for the moment. As none of us is ever separate from any other being, including the Supreme Being, we need to look at the social aspects of our lives together. As explained in *The Four Spheres*, what is happening to humanity at the present time is unprecedented in the fourteen billion years of evolutionary history. It is thus extremely difficult to foresee how events could unfold in the next few decades and on into the next century.

One of the greatest uncertainties is the way that the human population could change during the rest of this century, for the mathematics does not give a clear picture. We bring children into the world and they, in turn, have children. But what will the world be like when these grandchildren come to have children of their own, which in my case will be about 2040, shortly before my hundredth birthday, if I live that long. And how many people will be living on Earth in 2100, when my twin granddaughters will be ninety, having grandchildren and maybe great grandchildren of their own, in the normal course of events?

Well, the United Nations regularly publishes statistics on fertility and mortality rates in all countries in the world, which statisticians applying Bayesian probability formulae (introduced in *The Four Spheres*) use to project population growth into the future. Here is a diagram of their latest projections from September 2014, published in *Science*, showing that the human population has trebled in my lifetime. The solid red line denotes that the world population could rise to about 11 billion in 2100 based on the UN's 2012 figures. The shaded areas—dark and light, respectively—project ranges of 8.9–13.2 billion and 9.6–12.3 billion with 80% and 95% prediction intervals. The dashed blue lines indicate the previous UN high and low variants of around 16 and 7 billion, with a peak of 8–9 billion in the latter case.



So how relevant are such models of population growth to us in our daily lives, preoccupied, as we are, with families, jobs, and other projects? We make love to bring children into the world on the assumption that they, in their turn, will have children and so on indefinitely, continuing an evolutionary process that began with sexual reproduction about a billion years ago. We then go to work to earn a living to feed, clothe, and shelter our families on the assumption that the economy will continue functioning in the manner that it has done since the invention of money some 4,000 years ago and the birth of modern banking a few hundred years ago.

Well, we don't need to be too concerned about the figures, for the probability techniques used to calculate them are based on expectations from evolutionary history, assumptions that are no longer valid. The global economy does not provide people with financial security, as politicians would have us believe, because it is inherently unstable at these chaotic times and could self-destruct at any moment.

Most significantly, what the above chart shows is that even our most intimate moments are part of a dynamical system, which scientists and governmental officials study, to say nothing of geneticists and family historians. As systems theorists have discovered, the macroscopic behaviour of a system cannot necessarily be derived from its microscopic characteristics. So human population growth is a result of all evolutionary processes on Earth, which first needs to be seen in the context of the physical universe, called the hylosphere in *The Four Spheres*.

Nor is this all. As that book explains, the hylosphere is but a tiny part of the Cosmos as a whole, consisting of the Absolute, as Ultimate Reality, embracing the Numinosphere, noosphere, biosphere, and hylosphere, each nested within the previous sphere in the sequence, as the Great Chain of Being. So it is vitally important that we stop attempting to understand mathematics and psychology in the context of the laws of physics, which do not constitute the laws of the Universe. The Absolute, as Consciousness, provides the Cosmic Context and Gnostic Foundation for all our lives.

Most significantly, the Totality of Existence—as the ultimate system and macroscopic domain of discourse—is inherently stable, always in balance, following one, simple universal law. On the other hand, when we microscopically study narrower domains of discourse, such as the physical universe, evolution as a whole, the weather, population growth, or the psychodynamics of society, from couples to the entire population, we find a much more uncertain situation. So to live equanimously at these turbulent times, we can do no better than to rest in Wholeness.

In other words, to objectively study the growth and decline of human population, we need to stand outside ourselves, viewing our lives from a Holoramic 'Whole-seeing' vantage point, cognate with *panoramic* 'all-seeing'. Otherwise we get embroiled in emotional disturbances, not able to see our current situation clearly and dispassionately. This is essentially what Johannes Kepler did in developing the first two laws of planetary motion in *New Astronomy* in 1609, challenging the Aristotelian and Christian belief that the Earth is the centre of the solar system and hence the universe. For to calculate the orbit of the Earth around the Sun, Kepler imagined that he was standing on Mars, observing the Earth, showing that the Earth's orbit follows a similar elliptical path to the other five planets known at his time. Einstein said that this thought experiment, unknown to Galileo, was 'true genius'.

Looking at the psychodynamics of society as a whole, as evolution passes through its Accumulation Point, the predominant cultures of the world are blindly accelerating into chaos, while an awakening countercultural movement is emerging, focused on evolutionary convergence, marking the increasing polarization of society. But even if humanity survives this apocalyptic transitional process, does this mean that *Homo sapiens* is destined to live in love and peace for ever after? I ask this question before looking further at humanity's current situation because in mathematics it is often useful to set a problem within its boundaries. And as it is with mathematics, so it is with our lives together.

Well, according to the Principle of Unity—the most basic law of the Universe—birth and death and growth and decay are just two sides of the same coin; they cannot be separated. So eventually, a generation of children will be born who will not grow old enough to have children of their own. All structures in the Universe are born to die, without exception, including our bodies, Western civilization, and *Homo sapiens*. In the cases of the global economy and our species, these are crucial issues that have preoccupied all my thoughts since 1980 and 1982, respectively, when this awesome Cosmic vision appeared in consciousness.

As I can see today, humanity could survive for another 10^n years, where n could be 1, 2, 3, or even 9, a little less than the length of time in years before the Sun turns into a red giant and white dwarf in some five to six billion years time. Yet, from a Cosmic perspective, it makes little difference what the value of n might be. This number is minuscule within the overall scheme of things.

As described in *The Four Spheres*, John Leslie and Nick Bostrom—director of the Future of Humanity Institute—have calculated the relative probabilities of n = 2, 3, and 6 using Bayes' theorem, indicating that

the probability of a smaller value for *n* is higher than we would like to contemplate. For instance, in *The End of the World*, Leslie compares extinction by 2150 (*n* = 2.13) with the end of the human race in a thousand or million years' time (*n* = 3 or 6). His calculations give the probability of extinction within a few generations as 50% and 99.9%, respectively. The relative probabilities that *Homo sapiens* will become extinct in the next century or two is far higher than humanity will survive for a million years and about the same as it surviving for a thousand. As a US study in 2015 showed, Earth is entering a new mass extinction phase, with humankind being one of the most endangered species on the planet.

However, as far as I can tell, no one studying the many existential risks facing humanity today has recognized that evolution is currently passing through its Accumulation Point and is accelerating towards its Glorious Culmination, when a spiritual species of superintelligent, superconscious beings could be living in the eschatological Age of Light at the end of time in the Eternal Now.

As a corollary, few recognize that the greatest threat to our continuing existence as a thriving species does not come from outside us in the form of rapid climate change or peak oil, for instance. Rather, the greatest existential risk is psychospiritual, coming from our deluded and schizoid minds. Specifically, the greatest danger is our inability to adapt to our rapidly changing environment, which scientists and technologists are creating as the latest manifestation of some fourteen billion years of evolution.

In theory, we should be able to adapt to the changing circumstances in our lives, for humanity is the most adaptable of all the species. Using the metaphor of a computer, very few of our thoughts and actions are hard-wired. The innate instincts and automatic reflexes of babies to suck, grasp, cry, and respond to stimuli mostly disappear within the first few months of life. Our learning—corresponding to software and data in computers—mostly determines the way that we view the world and ourselves, and hence our behaviour. When we are in touch with our innate nature, we humans are more intuitive than instinctive.

However, because of hundreds and thousands of years of cultural conditioning, making changes to our reasoning that we urgently need to make is notoriously difficult. It is as if our learning has become hard-wired, laid down in the circuitry in the brain. Indeed, even this can be changed, for there is some evidence today that our bodies are not as immutable as was once believed. Mind changes can bring about somatic changes, even in our DNA molecules. But as belonging to groups is a central motivating factor, as Abraham Maslow pointed out in his hierarchy of needs, we shall only be able to make the changes that we urgently need to make at the cultural and social levels.

Vimala Thakar was one who well understood humanity's perilous predicament. In the opening paragraph of *Spirituality and Social Action: A Holistic Approach* in 1984 she wrote, "In a time when the survival of the human race is in question, continuing with the status quo is to cooperate with insanity, to contribute to chaos." She therefore asks, "Do we have the vitality to go beyond narrow, one-sided views of human life and to open ourselves to totality, wholeness?" For as she says, "The call of the hour is to move beyond the fragmentary, to awaken to total revolution."

Regarding humanity's ultimate destiny, of course, we would all like *n* to be as close to 3 as possible. 1,000 years sounds a reasonable length of time into the future, not of immediate concern to us today. Yet, looking backwards in time, it is less than a thousand years since William the Conqueror captured the English throne, due to be celebrated in 2066 in the normal course of events. And it is only two and a half millennia since the first great axial period, when Pythagoras, Socrates, Plato, Aristotle, Euclid, the Buddha, Laozi, Confucius and many others laid down the cognitive and spiritual foundations of the world we know today. We can still feel their influence and that of their successors, as evolution in the

noosphere has essentially been a cumulative process, building on what has previously been discovered, with an occasional change of direction on the way.

However, the concept of orbit in chaos theory tells us that we can no longer predicate the future on the past. Traditional forms of mathematics used in relativity and even quantum theory do not adequately map the complexity of the weather, ecosystems, or the psychodynamics of society. A simple example of an orbit is the sequence of integers 1, 2, 3, and so on, formed by this linear recurrence relation, with $x_0 = 0$:

$$x_{n+1} = x_n + 1$$

However, this quadratic recurrence relation, of similar form to the function that generates the beautiful Mandelbrot set, has quite different characteristics:

$$x_{n+1} = ax_n(1 - x_n)$$

This function is known as the discrete logistic map, which Robert May studied in the 1970s, later to become Chief Scientific Adviser to the UK Government and president of the Royal Society of London. This simple equation is derived from the continuous logistic equation, which Pierre François Verhulst used to model population growth constrained by physical resources in the 1840s. Here a denotes the rate of growth of a population of animals as 105% or 1.05, rather than 5% or 0.05, for instance. The initial values for this nonlinear difference equation (x_0) are the uncountable real numbers between 0 and 1, inclusive, which means that a needs to be between 0 and 4, inclusive.

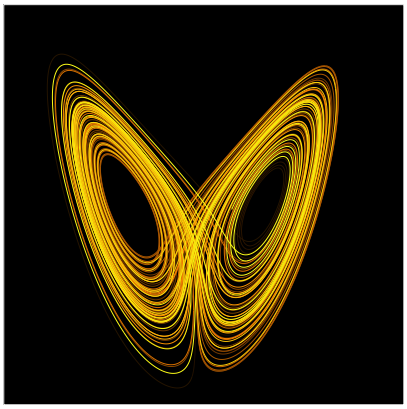
The set of values (x_0) are called the *basin of attraction* in chaos theory, setting out on an orbit towards an *attractor* as the function is iterated. However, to the amazement of the mathematicians who first studied such functions in the 1960s and 70s, the orbits that the values follow on their way towards their attractors are far from predictable. In some instances, they lead to deterministic chaos, where the attractor is a completely random set of values, of apparently no relationship to each other or to any previous set in the iterative process.

In the case of the logistic map, if $a \leq 1$, then the attractor is zero for all initial values of x_0 and the population steadily diminishes, becoming extinct. If $1 \leq a \leq 3$, then the orbit converges at different rates on a single value, denoting the maximum population for different rates of growth. For instance, if $a = 2$, then the attractor is ½. However, as a increases from 1 to 3, the orbit changes in character. Initially, it is monotonic for most values of x_0, steadily increasing or decreasing towards the attractor. But as a approaches 3, where the attractor is ⅔, the orbit becomes increasingly oscillatory, bifurcating as a grows larger than 3, oscillating between two values, as the attractor.

Then, as a continues to increase, the attractor splits into four, then eight, bifurcating at shorter and shorter intervals until a is about 3.569945672, called the *accumulation point* for the function. Paradoxically, this point occurs at a finite value, reached after an infinite number of bifurcations, a situation that greatly puzzled Zeno of Elea in the fifth century BCE. Today, we can explain such phenomena with the concept of limit in mathematics. The infinite sum of a series of geometric terms diminishing by a constant value converges on a finite value. For instance, if the constant is 2, then the finite sum is 2.

In the case of bifurcating systems in chaos theory, in the 1970s, Mitchell J. Feigenbaum discovered that the diminishing factor in any geometric series of bifurcations converges on a mathematical constant δ, which is about 4.669201609. This Feigenbaum constant is not just applicable to the logistic map. It is a universal constant, like π, applicable to all bifurcating systems, no matter what function generates them. Such mathematical constants are even more basic than the fundamental physical constants that apparently govern the physical universe, such as the speed of light and the number of spatial dimensions, briefly mentioned in *The Four Spheres*.

In the case of the logistic map, as a progresses beyond the accumulation point, the attractor becomes more and more chaotic, but with pools of order buried deep in the chaos, reflecting the entire map, like the self-similarity of fractal geometry. However, when $a = 4$, even these patterns seem to disappear. The attractor is a completely random set of values between 0 and 1, whose orbits also appear random, with no logical relationship between successive values. Furthermore, the orbits and the attractor are highly sensitive to the initial values x_0, unlike for smaller values for a, where the function converges on a single point or set of points. Systems theorists often use this sensitivity in their definitions of *chaos*.



This characteristic of chaos is best known to the general public as the butterfly effect, where a small perturbation in one part of a system could later have a large effect in another part, like a hurricane. Edward N. Lorenz coined this term in his study of weather systems, where the mathematics is somewhat more complex, but more ordered, leading to strange attractors following beautiful orbits, such as this.

So what has all this got to do with evolution, with our children and grandchildren's future happiness? Well, at a seminar in 2000, I learned from Nick Hoggard, a software developer living in southern Sweden, that the bifurcations of the logistic map could be applied to evolution as a whole. Each bifurcation corresponds to a major evolutionary turning point, such as the so-called origin of life on Earth, the birth of *Homo sapiens*, and the industrial revolution. As Nick discovered, the periods between successive evolutionary turning points are diminishing exponentially by the Feigenbaum constant, converging on evolution's Accumulation Point about 2003, give or take a couple of years. It is not possible to be entirely accurate because the input parameters are not precise.

This is a slightly different estimate than that of evolutionaries, like Barbara Marx Hubbard, who say, influenced by the geometric nature of the Mayan calendar, that evolution's Accumulation Point was reached at the winter solstice in 2012. And computer scientists like Victor Vinge and Ray Kurzweil are predicting what they call a technological singularity will happen during the 2020s, when machines with artificial intelligence will take over the workplace, making human beings redundant.

Despite these slight differences, it is crystal clear that by viewing the fourteen billion years of evolution as a single dynamical system, an infinite series of bifurcations is rapidly terminating at evolution's Accumulation Point, the most momentous turning point in its fourteen billion-year history. There are no more distinct evolutionary points to be discerned, as evolution leads us into the chaos we see in the world around us today.

To explain what this means, Nick used the metaphor of a dripping tap, which is another example of a bifurcating system converging at an accumulation point, when the tap is turned full on and flows continuously. In terms of evolution as a whole, there are no more distinct turning points to be discerned; the evolutionary tap has now opened up completely, giving us the wonderful opportunity to accelerate towards evolution's Glorious Culmination in the Divine, much as the mystical scientist Pierre Teilhard de Chardin prophesied in *The Human Phenomenon*, completed in 1940.

However, for the evolutionary tap to flow freely and uninhibitedly within us, there can be no blockages, such as those we inherit from the cultures we are born into or the subcultures we migrate to. Therein lies humanity's greatest challenge today. We are living on the edge of chaos, but because of our mechanistic,

cultural conditioning—accumulated over hundreds and thousands of years of human learning—we do not have the freedom to understand how to bring universal order into our lives.

Resolving this critical issue is not something any of us can do on our own or in elite, exclusive groups. We humans are like cells in the body politic, as the epigenetic biologist Bruce Lipton realized in a life-changing eureka moment in 1985. As he wrote in *The Biology of Belief*, "Just like a single cell, the character of our lives is determined not by our genes but by our responses to the environmental signals that propel life." We are all interdependent on each other for our health, well-being, and survival as a species. There are no piecemeal solutions, which Karl Popper favoured in *The Open Society and Its Enemies*. Only a holistic approach, which some regard as idealistic or Utopian, has any chance of success.

I am reminded here of the days in my youth when I would read the bridge column in my daily paper. One deal has influenced my life ever since. At a bridge tournament, one declarer realized that the only way he could make his contract was if the cards in one suit were divided 5-0 East-West, a 2% probability. Accordingly, he played his hand assuming this split and was the only one in the room to make his contract. That is how I live my life. I know that the more probable 3-2 and 4-1 breaks will not succeed. The only way to play the cards that I have been dealt in life is to assume almost impossible odds.

For our chances of success look pretty remote today. Our social environment inhibits us from flowing freely with the creative power of Life, preventing us from awakening from our slumbers, not able to adapt consciously to the unprecedented changes happening to humanity at the present time, driven by science and technology. If we are to play our cards correctly, we need to take risks, recognizing that the body politic—consisting of Western civilization and the other major cultures in the world, such as the Chinese, Russian, and Islamic—is fatally wounded, terminally ill with cancerous cells, to mix metaphors. For they are all bound together by the inherently unstable global monetary system, even though they have different political and economic ideologies.

This is a really tricky situation for we are all inseparable cells in the body politic, which need to synergistically pass through a metamorphosis of the entire infrastructure of society within a very short space of time, like caterpillars becoming free-flying, beautiful butterflies. For *metamorphosis* means 'change of form', from Latin *metamorphosis* 'transformation', from Greek *metamorphoun* 'to transform', from *meta-* 'change' and *morphē* 'shape, form'. In more technical terms, larvae become pupae, which become imagos or imagines 'perfect insects', a term also used in psychology to mean 'idealized image of a person, usually a parent, formed in childhood and persisting unconsciously into adulthood'.

But such a cultural transformation is not biological, like the metamorphosis of insects. As explained in *The Four Spheres*, the biological species of *Homo sapiens* has evolved into the noological species of *Homo noeticus*, which is evolving into *Homo divinus* today. Accordingly, the metamorphosis that humanity needs to go through is one of metanoia, from Greek *metanoia* 'repentance', from *metanoein* 'change one's mind or purpose' from *noein* 'to have mental perception', from *noos* 'mind, thought'.

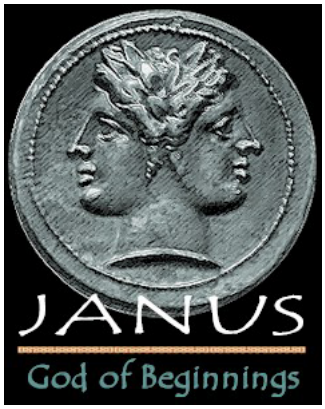
In 1873, in *Literature and Dogma*, Matthew Arnold thought that the word *metanoia* had been mistranslated as 'repentance' in the Bible. Arnold—a great uncle of Julian and Aldous Huxley, major figures in evolutionary history—said, "Of *metanoia*, as Jesus used the word, the lamenting one's sins was a small part; the main part was something more active and fruitful, the setting up an immense new inward movement for obtaining the rule of life. And *metanoia*, accordingly, is: *a change of the inner man*," a 'spiritual conversion'. This is the only way in which we can realize the cultural imago of the perfect society, following, for instance, the Buddha's Eight-fold Path.

Einstein said much the same thing at a Nobel anniversary dinner in December 1945: "The war is won, but the peace is not. The great powers, united in fighting, are now divided over peace settlements." Then in the *New York Times Magazine* in June 1946, in an article titled 'The Real Problem Is in the Hearts of Men', he said, "a new type of thinking is essential if mankind is to survive and move to higher levels. … Past thinking and methods did not prevent world wars. Future thinking *must* prevent wars."

Another who has written extensively on the human predicament is the social psychologist Erich Fromm, who in a series of brilliant books from 1942 to 1976 outlined a strategy by which our grievously sick society could be healed. Most importantly, in his greatest masterpiece *To Have or To Be?*, influenced by Meister Eckhart and Shakyamuni Buddha, he recognized that a new science of humanity would be needed for the art and science of social reconstruction. However, he was uncertain of success, saying,

> Whether such a change from the supremacy of natural science to a new social science will take place, nobody can tell. If it does, we might still have a chance for survival, but whether it will depends on one factor: how many brilliant, learned, disciplined, and caring men and women are attracted by the new challenge to the human mind.

At the heart of this new type of thinking is the transformation of conflict-ridden, either-or thinking into a harmonious, both-and way of life. While such a transformation is happening in oscillatory stages, it can only be fully realized through a spontaneous spiritual conversion. This is a little like the change that Sweden made at 05:00 on Sunday, 3rd September 1967. Before this date, Swedes had driven on the left-hand side of the road with left-hand-drive cars. But at that moment in time, they switched to the right-hand side, despite a referendum voting overwhelmingly against the change. Of course, this was more sensible, for when I visited Sweden for the first time in 1960, I was driven there from Norway, requiring my host to switch lanes on the border. Something similar is needed in society today.



The Cosmic Equation lies at the heart of the science of humanity that Fromm called for forty years ago, guiding every moment of my life since the mid 1980s, when I became aware of the coherent system of thought that was then emerging in consciousness. We can see that the ancients were intuitively aware that the Principle of Unity is the universal generating power from Janus, one of the oldest gods in the Roman pantheon, and from two-faced gods, who appeared repeatedly in Babylonian art. As the god of beginnings, Janus has given his name to January, at the beginning of the year. Janus is also the god of transitions, such as the global transition process that humanity is passing through at the moment, from pathogenic either-or ways of thinking and living, to a healthy both-and approach to life. For Janus, able to see both the past and future, symbolizes that the Principle of Unity is as much the Goal of life as its Origin. Our lives, like the Universe in its entirety, begin at the end and end at the beginning.

However, I am very well aware that this, the simplest of all ideas, is the most difficult to grasp, which is why it has remained hidden despite Heraclitus, Laozi, and other mystics revealing it over the years. The central problem here is psychological, relating to the sense of identity that governs our lives. When we look around the world, websites estimating daily population growth tell us that there are currently about 7.4 billion body-mind-soul organisms living on our beautiful planet Earth, increasing by over 200,000 every day. All these human beings appear to be separate from each other, for we inherit identifying names from the families we are born into, our parents giving us other names to differentiate us from other family members, and governments issue unique social-security numbers at birth.

Apart from our names and numbers—as signs and symbols—we become identified with the colour of our skins, with the sex of our bodies, and with various religious, national, and other cultural institutions, for instance. This sense of a separate identity begins in infancy. But when we reach about seven years, a marked change happens in our development, illustrated with a well-known psychological test.

An infant is first shown a card painted yellow on one side and blue on the other. Then the card is held in front of the infant so that she or he can see only the blue side, with the yellow side facing the tester. The tester then asks the infant, "What colour can I see?" At six years of age, the infant generally answers 'blue'. She or he cannot see the other's perspective. Yet at about eight years of age, the answer is 'yellow'. The infant has grown into childhood, able to see both sides of the card with the mind's eye, even when one side is hidden from the physical senses.

Yet, how many of us are able to see both sides of any situation with Self-reflective Intelligence? How many are able to stand outside ourselves, caring for our entire species, carrying the joys and suffering of all beings on Earth? To attempt to do so can be so overwhelming that we cannot even manage our own affairs, as we see in the well-known psychological effect of codependency. Yet, as Andrew Cohen pointed out in *Freedom Has No History* in 1997, each of us, as individuals, need to take responsibility for the evolution of the entire human race. As he says, "To succeed, we must be prepared to do battle with the powerful conditioning, conscious and unconscious, of the whole race. That means we have to come out from the shadows and be seen. Like Atlas, we have to be willing to hold up the whole world on our shoulders. It's an awesome task."

Our inability to do so is the root cause of all the conflicts and suffering that have bedevilled human affairs and relationships for thousands of years. Our political and economic systems are based on this incapacity, on the false belief that we humans are separate from each other, that the 'other' is, in some sense, the 'enemy'. We see this most clearly in the Holy wars—wars about the Whole—that have been fought between the religions over the years, regarding scientism, atheism, secularism, and nonspiritual humanism as religions.

So even though the Principle of Unity represents two sides of a coin, which can be grasped in a hand, we constantly attempt to separate opposites, by categorizing beings as *A* or not-*A*. As another example, we have categorized our subspecies as *Homo sapiens sapiens* 'wise-wise human', distinct from *Pan troglodytes*, the chimpanzee, our closest relative, and the other species.

Accordingly, in 1973, Brandon Carter proposed that we humans have evolved to where we are today as the result of an 'anthropic principle' underlying the laws of physics, counteracting Hermann Bondi's 'Copernican principle' from 1952. For while "the Earth is not in a central, specially favoured position," as Bondi pointed out, "our situation is … necessarily privileged to some extent," as Carter observed. But this is not an either-or situation; it is a both-and one, which Richard Gott III aptly calls the 'Copernican anthropic principle'.

As it is with the species, so it is with us, as individuals. We are all unique beings, following our own particular journeys in life, having some similarities with others. For instance, I am a white male, a father and grandfather, born in England but resident in Sweden. Yet such categories do not determine who I am, what my True Nature and Genuine Identity might be. To realize my Authentic Self, I have needed to transcend the categories, realizing that none of us is ever separate from the Divine for an instant, when and where there is no 'I', 'me', 'my', or 'mine' separate from a 'we', 'you', 'he', 'she', 'it', or 'they'.

By the Principle of Unity, we thus all have two, inseparable identities in life: the mystical and mundane, from Latin *mundus* 'world'. The word *identity* derives from Latin *idem* 'same', meaning a

quality that remains the same throughout a person's life. This is usually taken to be our unique, human identities. But we all share Wholeness as our Genuine Identity, enabling us to view the Totality of Existence from a Holoramic 'Whole-seeing' perspective.

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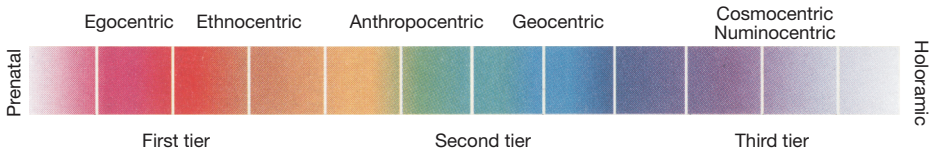
To see how evolution could help us to unify the mystical and mundane aspects of our identities, it is most useful to look at Ken Wilber's spectrum of consciousness.



This spectrum depicts how the cards have been dealt among the seven billion cells or souls in the body politic. As outlined in *The Four Spheres*, people living with first-tier levels of consciousness determine the rigid rules that govern our lives, while those living in the second-tier are beginning to pull away from the mainstream, as liberals rather than conservatives in the political spectrum. But this is still an either-or situation, not the both-and bipartisan approach we need for transformational harmony, a term that Ananta Kumar Giri, Vice-President of the Global Harmony Association, has coined in the spirit of compassionate confrontation.

Furthermore, the changes happening in society are taking place at too slow a pace to ensure that we pass through evolution's Accumulation Point relatively unscathed. Something far more radical is needed. As Jean Houston, a pioneering leader of the Human Potential Movement, has said, "Ours is an era of quantum change, the most radical deconstruction and reconstruction the world has seen."

So could society as a whole pass through such a psychospiritual metamorphosis between now and 2020, or at least by 2030? If not, it will be too late, as machines with so-called artificial intelligence will take over more and more jobs and the global economy will come crashing down with devastating consequences. What gives me hope that such a sudden transformation of culture and consciousness in society as a whole is possible is that many people have experienced such instantaneous awakenings, albeit sometimes just glimpses, not sustainable. Nevertheless, such brief insights are often life-changing, taking people in quite new directions in their lives.

Another reason for hope is René Thom's catastrophe theory in topological mathematics, introduced in 1975 in *Structural Stability and Morphogenesis: An Outline of a General Theory of Models*, as a translation of his 1972 book in French. For Thom showed that change in physical and biological systems—equally applicable in the noosphere—can be both gradual and abrupt. And in the latter case, catastrophes can be both breakdowns and breakthroughs.

As the status quo is no longer a viable option for humanity, we need to expect the unexpected. This is not social engineering in Popper's sense of this term, for the changes that need to happen in society can only come about from within, from the Divine Source of Life. Such a radical transformation of culture and consciousness is not mechanistic. As we are all the products of some fourteen billions years of evolution, we are not masters of our destiny, which is a characteristic of Popper's social engineer. Under these circumstances, all we can really do is accept 'what is', trusting that where Life is taking us all is what is meant to happen, no matter what our egoic or anthropocentric preferences and opinions might be.

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For the moment, that is enough on the psychosocial issues that arise from the nonlinear mathematics

of dynamical systems expressed through chaos theory. To see the Big Picture of our lives together, we also need to take a Cosmic perspective. Since the middle of the nineteenth century, such an all-encompassing perspective had been called a *worldview* or *Weltanschauung*, which I prefer. For *Weltanschauung* is derived from German *Welt* 'world' and *Anschauung* 'view', from Middle High German *anschouwunge* 'observation, mystical contemplation'. So *Weltanschauung* has a deeper meaning than *worldview*, indicating both scientific observation and spiritual meditation. It is through such a *Weltanschauung* that we can develop a comprehensive model of the psychodynamics of the whole of society, helping us to understand why we all behave in the way that we do.

In our interdependent world, what we need here is a cosmology of cosmologies that integrates all cultural cosmologies that have evolved during the millennia, showing how they all fit together as a coherent whole. For we can no longer afford to have holy wars—wars about the Whole—between different cosmologies. *The Four Spheres* provides a reasonably detailed description of such an all-embracing *Weltanschauung*. But for those who haven't read any of my writings before and to remind those who have, to further set the scene for the mathematics in the main body of this book, let us spend a moment on some of the key characteristics of this *Weltanschauung*.

As the use of this word indicates, the changes that we need to make if we are to intelligently adapt to our rapidly changing times are far greater than the transformation of the geocentric worldview into a heliocentric one that Copernicus, Kepler, Galileo, and Newton made from 1543 to 1687. Darwin, Einstein, and Bohm subsequently introduced further changes to the scientific worldview, exposing some of the cracks in materialism and mechanism, patching them over as best as they could.

But now we need to cocreate a revolution in science far greater than all others in human history, as some are aware. For instance, in 1986, Willis Harman, then president of the Institute of Noetic Sciences (IONS), founded by the late Edgar Mitchell, the sixth man to walk on the Moon, spoke these words at a new paradigm conference:

> Most educated people in this country [the USA] would think it pretty preposterous to suggest that the change that is taking place is at as deep a level as the change that took place during the Scientific Revolution, because that would imply, of course, that the near future—the early part of the next century—would be as different from present times as present times are from the Middle Ages.

In a similar fashion, in 1984, F. David Peat, Bohm's biographer, and John Briggs wrote in *Looking Glass Universe: The Emerging Science of Wholeness*, today's revolution in science "could bring us a world as removed from our current modern science as our science is from the occult certainties of the Middle Ages". Furthermore, as they said, this final revolution in science is likely to take only decades, in contrast to the hundreds of years it took for the first revolution to unfold.

What this means is that to live in love, peace, and harmony with the fundamental law of the Universe, like Alice, we need to pass through a looking glass into the weird and wonderful world on the other side, where we can play with paradoxes and self-contradictions to our heart's content, rather than rejecting and denying them, as we are encouraged to do by mathematics, logic, science, business, politics, and philosophy today.

We can see why paradoxes are so called from the Greek root *paradoxos* 'unexpected, incredible, marvellous; strange, startling', from *para-* 'contrary to' and *doxa* 'opinion, expectation'. So the original

meaning of paradox in the 1500s was 'A statement or tenet contrary to received opinion or belief; often with the implication that it is marvellous or incredible; sometimes with unfavourable connotation, as being discordant with what is held to be established truth, and hence absurd or fantastic; sometimes with favourable connotation, as a correction of vulgar error'.

Accordingly, in *Budget of Paradoxes*, published posthumously in 1872, the mathematician Augustus De Morgan called those who bring forward ideas counter to universal opinion 'paradoxers', who can also be called 'outsiders', such as Heraclitus in ancient Greece, who was called the 'Obscure' by his contemporaries. As the Law of Contradiction lies at the core of Western thought, society regards anyone who wishes to live in harmony with the fundamental law of the Universe a paradoxer, even though this word is rarely used.

We thus face a rather tricky situation, for the Law of Contradiction rejects self-contradictions from our reasoning, grossly distorting the cognitive maps and conceptual models that create our reality and guide our behaviour. For paradoxes abound in the world we live in, including mathematics. So, if we reject them because they make us feel uncomfortable or for any other reason, we must inevitably live in delusion. One reason why mathematicians reject contradictions from their reasoning is well illustrated by this little anecdote:

> The analyst G. H. Hardy once made this remark at dinner, and was asked by a sceptic to justify it: 'Given that $2 + 2 = 5$, prove that McTaggart is the Pope'. Hardy thought briefly, and replied, 'We know that $2 + 2 = 4$, so that $5 = 4$. Subtracting 3 we get $2 = 1$. McTaggart and the Pope are two, hence McTaggart and the Pope are one.

Just so. Not only are McTaggart and the Pope one from the perspective of Wholeness, our entire species is one, single, indivisible being, as is the Totality of Existence. What look like separate beings from a superficial perspective are just appearances in Consciousness, viewed as a vast Ocean, an infinitely dimensional ball of water. Indeed, everything that we conceive in the relativistic world of form is an abstraction from Consciousness, called *māyā* 'deception, illusion, appearance' in the East, probably from Sanskrit *mā* 'to measure', cognate with *measure* and *dimension*. In another Sanskrit word, the entire manifest world is *līlā*, the delightful play of the Divine.

Following the publication of Thomas S. Kuhn's *The Structure of Scientific Revolutions* in 1962, many refer to the changes currently being made in the scientific worldview a paradigm shift. For instance, the journal of the Institute of Noetic Sciences changed its name to *Shift: At the Frontiers of Consciousness* at the beginning of this century. And Stephen Dinan, formerly IONS Director of Membership and Marketing, has founded the Shift Network with the motto 'Accelerating the Next Evolution'. But what is happening in the world today is far more radical than a paradigm shift, a term that Kuhn used just six times in his book. In contrast, he used the term *paradigm change* twenty-three times, as a search of the Kindle edition quickly indicates.

This is a far more appropriate term for the transformation of either-or into both-and thinking. For *paradigm* derives from Late Latin *paradigma* 'pattern, exemplar, example'. And *pattern* derives from Middle English *patron* 'something serving as a model', from Medieval Latin *patrānus*, from Latin *pater* 'father'. When *pattern* entered the English language in the fourteenth and fifteenth centuries, it meant 'outline, plan, model, pattern' and 'model of behaviour, exemplar'. It acquired these senses because a patron had the sense 'one who commissions work', giving examples of what workers were required to copy. But the word could also be used as a human or role model.

We humans are essentially pattern-seeking and symbol-making creatures, constantly generating patterns, not only to bring a sense of order to our lives, but also for their aesthetic value. As we saw in the Humanistic Renaissance, art and mathematics meet in painting and music and they are meeting today in

mathematics itself in the beauty of fractals and sacred geometry. But even when pure mathematics cannot be outwardly expressed, there is profound beauty in the abstract patterns in mathematics, which G. H. Hardy celebrated in *A Mathematician's Apology* in 1940.

In mathematical terms, there is nothing more beautiful than the all-embracing Cosmic Equation. So to complete the paradigm change currently taking place in scientific and spiritual circles, we need to transform the Law of Contradiction into the Principle of Unity, recognizing that none of us is ever separate from anyone else for an instant.

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Now while the Principle of Unity is an irrefutable, universal truth, it is not the Absolute Truth, for the Truth that sets us free lies beyond all categories of thought. Another word for Truth is Consciousness, which is Ultimate Reality. But this does not mean that recognizing and realizing this fundamental principle of existence is a paradigm change from the prevailing scientific worldview, as some are claiming today.

The Absolute is beyond time, even beyond the distinction between time and Timelessness and that between form and Formlessness. Yet, as mystics have discovered through the ages, we can know the Truth with Absolute Certainty, as Gnostics or Jñānis, both words cognate with *know*. In Sanskrit, this view of the Cosmos can be eloquently denoted by *Satchitānanda* 'Bliss of Absolute Truth and Consciousness', cognate with Mohandas Gandhi's *Satyagraha* 'Truth force'. In popular parlance, *Satchitānanda* is *Nirvāna* 'extinction (of the separate self)'.

In other words, as Ramesh S. Balsekar, a former President of the Bank of India and an Advaita sage, has said, *Consciousness is all there is*. For *Consciousness* means 'knowing together', from Latin *cum* 'together with' and *scīre* 'to know', also the root of *science*. So when we learn to integrate all knowledge into a coherent whole, individual consciousness deepens and expands to such an extent that it becomes coterminous with Consciousness itself. Consciousness thus unifies the concepts of God and Universe, which did not make any sense to me as a schoolboy in the 1950s, deeply satisfying.

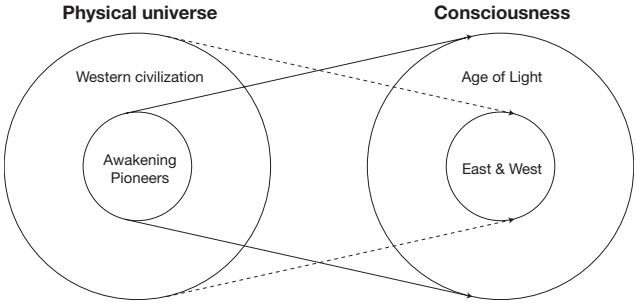
We have now arrived at evolution's Glorious Culmination at the end of time in the Eternal Now. From this Holoramic vantage point, we can see that the multiverse of physical universes and their components, including the brain, emerge from Consciousness; all beings in the manifest Universe are related to each other, never separate from God, Nature, or any other being for an instant. This is what I call the mystical worldview, described thus:

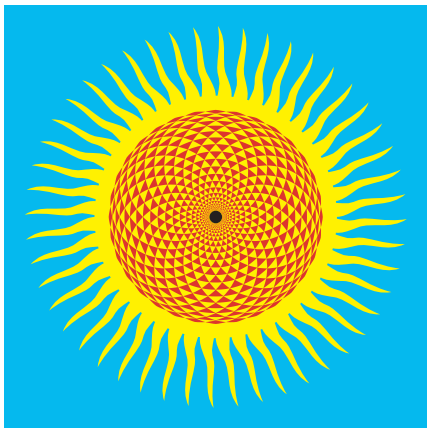
> It is from the Formless Absolute—as the Divine Datum of the Cosmos—that the entire relativistic world of form emerges, like waves and currents on and beneath the surface of an ocean, never separate from the ocean itself. This union of form and Formlessness is the Ocean of Consciousness, the centre of which is Love, the Divine Essence we all share, providing the Cosmic Context for all beings in the Universe, including all of us human beings.

Viewing the Totality of Existence as a dynamical system from this macroscopic perspective, we can see that the Great Attractor of all evolutionary processes on Earth is the Omega Point, as the Cosmic Christ or Buddha, inseparable from the Alpha Point, as the Basin of Attraction for all dynamical systems in the Cosmos. As Teilhard foresaw and experienced as a scientific mystic, when Alpha and Omega become unified in Wholeness, the personal becomes the impersonal and all there is is Love, which has no opposite. Yet this is not really a becoming. No one can return Home to Wholeness for nobody has ever left Home. We can only *be* Love, not become it.

So while transforming the Law of Contradiction into the Principle of Unity can be called a paradigm change, transforming the materialistic, mechanistic scientific worldview into a *Weltanschauung* that

recognizes Consciousness as Ultimate Reality is really a contextual inversion, as this diagram illustrates.





As shown here, humanity is currently emerging from the Dark Ages of Western civilization, entering the eschatological Age of Light. We can depict this cosmology with this diagram of the coherent light of Consciousness, which I programmed in Postscript in the 1980s and 90s, having adapted it from *Energy, Matter and Form*, published by the University of the Trees in 1979. As the diagram illustrates, Consciousness radiates from a black hole, a Singularity at the heart of the Cosmos, at its Divine Origin. But this light is not like the diffuse light of a light bulb or the Sun. Rather, this brilliant light is like the coherent light of a laser beam, enabling us to create and view a holographic map of the Universe, displaying its property of self-similarity, like fractals.

Of course, if sceptical scientists are to accept the validity of this mystical cosmology of cosmologies, it needs to be described in the most rational and systemic manner possible. Such a coherent way of thinking marks a radical change from the way that mathematicians usually model changing phenomena, for they mostly do so quantitatively, using such tools as differential and difference equations.

To study how and why we think and behave as we do, we need a more meaningful, qualitative approach, generalizing the abstract modelling methods that information systems architects use to build applications and databases in the Internet. For if such architects are to develop business systems adapted to the needs of the people, they need to know how the psyche works to its utmost depth and breadth. This is an understanding that is acquired through introspection and self-inquiry, by looking inwards rather than outwards, the primary focus of science and business today. It is in such a way that we can understand what is causing scientists and technologists to drive the pace of change in society to accelerate exponentially.

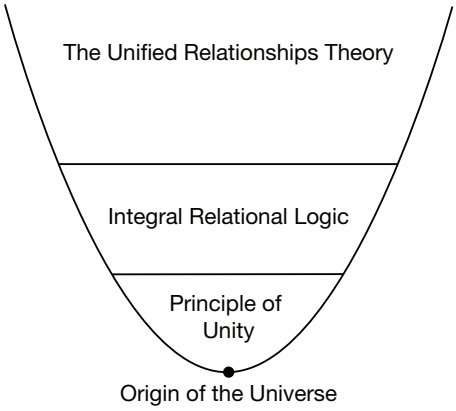
So let us look briefly at how the paradoxical Principle of Unity is encapsulated in a nonlinear, holographic system of thought, which I call *Integral Relational Logic* (IRL). In essence, Integral Relational Logic applies Self-reflective Intelligence to unify mystical, depth psychology and nonlinear mathematical logic—as sciences of the mind. It is therefore the solution to the business management problem that I was struggling to solve during the winter of 1980, as mentioned above.

More generally, Integral Relational Logic is the commonsensical art and science of thought and consciousness that we all implicitly use everyday to form concepts and organize our ideas in mathematical relations or tables and mathematical graphs or semantic networks. As this universal system of reason has evolved from the thought experiment already outlined, IRL is the IOS that manages all other Integral Operating Systems, in Ken Wilber's terms, defined in *Integral Spirituality*.

In the context of this essay, the most important feature of the semantic models in Integral Relational

Logic is that they have the property of self-similarity, like fractals. This characteristic arises directly from the modelling methods that underlie the Internet, which are of great abstraction and generality—even more so than mathematical category theory, first-order predicate logic, and abstract and universal algebra—applicable in all cultures, industries, and disciplines. If this were not the case, the Internet could neither exist nor expand at the hyperexponential rates we have been experiencing for the past two or three decades. So like the Internet, we can see that *the underlying structure of the Cosmos is an infinitely dimensional network of hierarchical relationships*.

As everyone on Earth as been implicitly using Integral Relational Logic throughout the entire history of human leaning, it provides the Cosmic Context, Gnostic Foundation, and coordinating framework for the *Unified Relationships Theory* (URT) or *Panosophy*, the complete synthesis of science, philosophy, and religion and of all the sciences and humanities. *Panosophy* derives from Greek *pan* 'all' and *sophia* 'wisdom', the fulfilment of what John Amos Comenius, the 'father of modern education', called *Pansophia* 'the Utopian dream of science', in the mid 1600s. Transcultural, transdisciplinary Panosophy thus enables us to see the Big Picture, answering such questions as "Who are we?", "Where have we come from?", and "Where are we heading?"



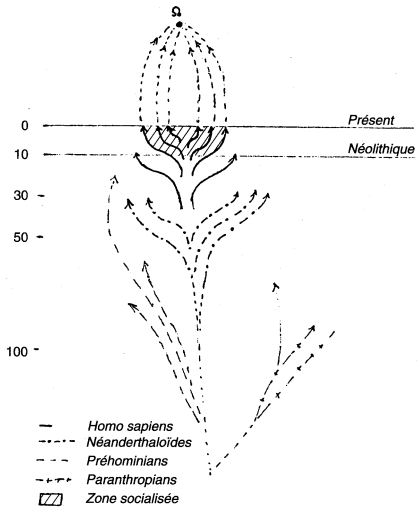
The relationship of the Principle of Unity, Integral Relational Logic, and the Unified Relationships Theory to the Origin of the Universe is depicted in this diagram. The Principle of Unity emerges directly from the Formless Absolute, as the first form to be created, from which all other structures evolve through a complex process of bifurcation. This gives rise to Integral Relational Logic, as an ontological and epistemological map of the Cosmos, corresponding to the abstract modelling methods that information systems architects use to build the Internet. In turn, IRL provides the Contextual Foundation and framework for the entire world of learning, which corresponds to all content on the Internet, in all books that have ever been published or ever will be, and all ideas and concepts in the consciousness of all humans everywhere in all time.

Most significantly, from the perspective of Western civilization, this diagram shows that Integral Relational Logic, as the psychological art and science of consciousness, is the primary science, the foundation for all the sciences and humanities. Mathematics and physics are but two specialist disciplines in transdisciplinary Panosophy, not special in any way. So if we are to intelligently adapt to the accelerating pace of evolution being driven by scientists and technologists, we need to stop treating physics as the primary science. In recent years, biology, as the science of life, has made some attempts to usurp the throne. But with so-called natural scientists unwilling to admit Life and the Truth into science, we can only be lead into delusion if we follow their lead.

In the context of this essay, this diagram also shows that the Cosmic Equation lies beneath the foundations of mathematics, which have been in a quagmire for over a century. It is advisory not to venture into this morass too far, for we very quickly become bogged down in the obfuscation. On the other hand, when applied to physics, we can see that the Cosmic Equation is the equation that Albert Einstein sought for at the core of his unified field theory and the one, simple, elegant equation that can explain everything at the heart of what Stephen Hawking calls the theory of everything.

This depiction of all knowledge is open-ended because from the perspective of Formless Wholeness, there are no limits to the details that can be discovered in the Totality of Existence, which is the most

complex structure in the Cosmos. However, because we live in a holographic Universe possessing the property of self-similarity, the details don't really matter. For, in Reality, all bifurcating streams of evolution converge in Ineffable Wholeness at the Omega Point of evolution.



Here is a diagram that Teilhard drew of how the final few years of this bifurcating universal process could reach its Glorious Culmination in human consciousness. It is not to scale, for while it begins at the birth of *Homo sapiens* about 200,000 years ago, by analogy to other living layers, Teilhard estimated the time to the Omega Point to be in millions of years, Sarah Appleton-Weber tells us. He was not to know that the Omega Point would be reached within an individual born during his lifetime and that it could be reached in society as a whole within the next generation or two. Indeed, this is absolutely essential if our grandchildren are to survive to the end of the century.

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What Teilhard's diagram also illustrates is that evolution has been more divergent than convergent throughout the entire history of human learning, leading to a pandemic of fragmentation, the central problem of our times. Indeed, this is not a new phenomenon. Bifurcating evolutionary processes have been more divergent than convergent throughout the entire history of evolution since the most recent big bang, as we see in the galaxies of galaxies, the periodic table, and the beauty of the diversity of the species all around us.

But in psychosocial terms, fragmentation leads to delusion. As Erich Fromm and J. Krishnamurti, among others, have pointed out, we live in a sick society, which can only be healed when our fragmented minds are healed by studying transcultural, transdisciplinary Panosophy. David Bohm highlighted the consequences of this ubiquitous malaise in the opening paragraph of *Wholeness and the Implicate Order* in 1980:

> Fragmentation is now very widespread, not only throughout society, but also in each individual; and this is leading to a kind of general confusion of the mind, which creates an endless series of problems and interferes with our clarity of perception so seriously as to prevent us from being able to solve most of them.

One consequence of our fragmented minds is that Western civilization, in particular, is based on seven pillars of unwisdom, a term that Arthur Koestler introduced in *The Ghost in the Machine*. As mentioned in *The Four Spheres*, these are misconceptions of God, Universe, Life, humanity, money, justice, and reason, leading to widespread delusion and existential fear.

As chaos theory shows that the infinite series of these bifurcations is now terminating in evolution's Accumulation Point, we thus have the fantastic opportunity to harmonize evolutionary convergence, rebuilding society on the seven pillars of wisdom, recognizing that none of us is ever separate from the Divine, Nature, or any other being for an instant.

In this respect, we can learn much from a key feature of the information systems architect's job in business. In the coherent conceptual models that such systems designers build of business systems, all classes of object are measured in exactly the same way, whether quantitatively or qualitatively. Similarly, in transcultural, transdisciplinary Integral Relational Logic, all concepts are formed by paying careful attention to the similarities and differences in the data patterns of experience. In other words,

mathematical concepts, like numbers, curves, and groups, are treated in exactly the same way as money, mass, space, time, Universe, God, humanity, self, ego, and so on.

Bohm also used this commonsensical holistic approach to scientific inquiry to unify relativity and quantum theories, a notion of order that the artist Charles Biederman had given him. Then in 2005, Chris Clarke, a professor of mathematical physics in England, suggested in an email that the egalitarian way of treating all beings in the same way in Integral Relational Logic could be called 'radical egalitarianism'—very original.

However, for the most part, we do not treat all concepts in exactly the same way in our cognitive maps, leading us into delusion. Most significantly, we regard our body-mind-soul organisms and our relationships to other similar beings as special. Yet neither we as individuals nor as a species are special. We are subject to exactly the same laws of the Universe as all other beings.

Furthermore, it is important to note that the Unified Relationships Theory is not contained in symbols on the pages of this essay. For theories do not exist 'out there' in printed and electronic form. As Bohm pointed out, "The word *theory* derives from the Greek *theoria*, which has the same root as *theatre*, in a word meaning 'to view' or 'to make a spectacle'. Thus it might be said that a theory is primarily a form of *insight*, i. e. a way of looking at the world, and not a form of *knowledge* of how the world is."

Einstein well understood this distinction, as he explained in 1945 in a letter to Jaques Hadamard, who was then conducting a survey of some of the leading mathematicians of his day into how they develop their ideas. Einstein wrote, "The words or the language, as they are written or spoken, do not seem to play any role in my mechanism (sic) of thought. The physical entities (sic) which seem to serve as elements in thought are certain signs and more or less clear images which can be 'voluntarily' reproduced and combined." He then went on to say that this combinatory play in the mind occurs "before there is any connection with logical construction in words or other kinds of signs which can be communicated to others".

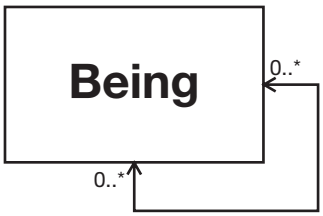
This is an incredibly important point, for it explains why machines, like computers, can never program themselves to develop artificial general intelligence, exceeding any level of intelligence that we humans have the potential to realize. Using the meaning triangle, defined in *The Four Spheres*, computers need explicit signs to denote both concepts and the signifiers that represent them. On the other hand, concepts in our minds do not require such symbols. They exist as mental images, grounded and embraced by Consciousness, beyond all categories, including the analytical mind. This can best be understood with what Aurobindo called the Supermind, which "starts from unity, not division; it is primarily comprehensive, differentiation is only its secondary act."

What this means is that people with a reasonable degree of mystical awareness can intuitively experience and visualize the Universe we all share, often expressed in beautiful music, poems, and paintings. However, I am more a rational artist, having been educated and trained in mathematics and computer science and having worked as an information systems architect for most of my business career. As such, I am a generalist, specializing in abstract thought, rather like my doctor, who calls herself 'specialist in general medicine'.

As evolutionary divergence has led to a multitude of diverse cultures around the world, this spreading out process has also led to a polyphony of languages to represent their different worldviews. This situation makes communications rather strained because we need to empathize with other people's fragmented

mindsets to understand what they are saying. We can resolve this rather tricky situation when we recognize that at the heart of the universal *Weltanschauung* that we all share lies the ancient, eternal wisdom that Isaac Newton and Gottfried Leibniz—the discoverers of the infinitesimal calculus—sought for as *prisca sapientia* and *philosophia perennis* in the 1600s.

Integral Relational Logic—as the universal art and science of thought and consciousness—also provides the transcultural language that underlies all the natural languages in the world, the most widely spoken by native speakers being Mandarin, Spanish, English, Hindi, and Arabic, categorized in several language families, such as Sino-Tibetan, Indo-European, and Afro-Asiatic.



At its heart, Integral Relational Logic is a very simple language, consisting of just one word and diagram in the notation of the Unified Modeling Language (UML), developed at Rational Software in the 1990s, now a subsidiary of IBM, my former employer. What this diagram shows is that all beings in the Cosmos are related to all others, including themselves, in zero to many different ways, some of which can be classified and some of which defy categorization and must remain a mystery.

In other words, in Reality, the Totality of Existence is a single, indivisible being, providing the unifying Cosmic Context and Gnostic Foundation for our species, also viewed as an entire whole, with no divisions or borders anywhere. Despite what we are taught in religion, science, and business, no one is ever separate from the Divine, Nature, or any other being for an instant.

The language of Integral Relational Logic then expands in further diagrams using mathematical relations as nodes in mathematical graphs to form a single semantic network mapping the Universe. This shows that the Cosmos is maintained in perpetual balance by the paradoxical Principle of Unity. There are no more than a couple of dozen words in Integral Relational Logic, the most important of which are *Datum*, *being*, *set*, *class*, *entity*, *attribute*, *structure*, *form*, *relationship*, and *meaning*, using the English language to represent them. These are the primal concepts that we need to pull ourselves up by our bootstraps, described in full in *Integral Relational Logic*, Part I of a 1500-page trilogy on *Wholeness*.

From a historical perspective, the language of Integral Relational Logic enables us to realize Comenius' dream of a Pansophic College—as an Academy of Universal Wisdom and Light—which did not take off in London in 1642. For John Wilkins, leader of the Oxford Experimental Science Club, wanted to have nothing do with Pansophia when he and others came to set up the Royal Society of London for Improving Natural Knowledge in the early 1660s, leading to a deep split between science and spirituality.

Yet, shortly after the Royal Society was established, it commissioned Wilkins to create a universal language for its rather narrow, superficial view of science in 1668. As Melvyn Bragg tells us in *The Adventure of English*, "Wilkins argued that since the minds of everyone functioned in the same way and had a similar 'apprehension of things', there was no reason to believe there could not be one universal language. This language would not only make international cooperation on every level simpler than ever before, it would also 'prove the shortest and plainest way for the attainment of real knowledge, that has yet been offered to the world'."

John Locke went even further in *An Essay Concerning Human Understanding* in 1690. He wrote that the greatest part of the disputes in the world could be resolved if the simple ideas underlying words were understood. This distinction between words and the concepts that they represent lies at the heart of semiotics, which Ferdinand de Saussure and Charles Sanders Peirce founded around the turn of the twentieth century, encapsulated in the meaning triangle.

However, although Integral Relational Logic is the universal language we all share, I write in English, my native tongue. English has had a long evolutionary history since the Angles, Saxons, and Jutes invaded England in the fifth century after the Romans left. It is one of the richest languages in the world, with a comparatively simple syntax, assimilating words from most of the other European languages and many from those farther afield.

There is just one major difficulty with English. It has evolved over the years to represent a fragmented worldview that is very far removed from Reality. To address this problem, David Bohm suggested to me in the mid 1980s that we need to study the archaeology of language, going to the roots of words before their meanings were distorted by the Western mind, wherever possible to their Proto-Indo-European origins some six to seven thousand years ago.

As just a couple of examples, to develop a comprehensive model of our rapidly changing world, I have needed to change the meanings of *science* and *Universe*, as they are generally understood today. By *science* I mean the systemic study of the relativistic world of form in its entirety—both its inner psychospiritual and outer material aspects. And by *Universe*, I mean that these inquiries are conducted within and on the Cosmic Context and Gnostic Foundation of the Formless Absolute, which embraces the entire world of form as Wholeness, experienced as Consciousness. More examples are in the evolving Glossary on my website and in Part I of the *Wholeness* trilogy, yet to be updated with other terms scattered among my recent writings.

Another solution to this linguistic problem is to use mathematics, as the language of science. As Galileo wrote in *The Assayer* (*Il Saggiatore*) in 1623, "[Natural] philosophy … is written in the language of mathematics, and its characters are triangles, circles, and other geometric figures without which it is humanly impossible to understand a single word of it; without these, one wanders about in a dark labyrinth."

Since then, mathematics has moved on considerably, not the least in the general theory of relativity and fractal geometry, which shows that we can no longer map the natural world around us through traditional shapes in Euclidean space. Furthermore, when we learn to map the Cosmic Psyche, we need to examine more carefully the role that mathematics plays in our studies. In the case of Panosophy, while *The Four Spheres* describes the coherent cognitive map and conceptual model that we can use to make complete sense of all our experiences—from the mystical to the mundane—it does so with limited mathematical explanation. This essay rectifies this omission in order to complete the final revolution in science that has been unfolding for the past few decades.

For the abstract semantic modelling methods underlying the Internet are not the only transcultural and transdisciplinary languages in the world today. Ever since the ancient Greeks, pure mathematics has provided a similar role in scientific inquiry. In the context of this essay, because the holographic Universe possesses the property of self-similarity, like fractals, the mathematics involved is essentially the same for all dynamical systems, from dripping taps, through population growth and weather forecasting, to evolution as a whole.

There is one more cosmological issue that we need to address before looking that how we could collectively deal with the practicalities of evolution's unprecedented rate of change. For the past few decades, a war has been raging between neo-Darwinists and Creationists over the issue of intelligent design. The former assert that there is no designer of the Universe and hence of evolution and hence of us.

In contrast, the latter believe that God created the universe in just six days, as it says in Genesis, and rested on the seventh day.

We can reconcile this dichotomy when evolution becomes fully conscious of itself within us. By applying the Principle of Unity, we realize that we humans are never separate from the Divine for an instant. In other words, while the Universe is intelligently designed, there is no separate designer thereof.

As an example of this fundamental creative principle, Jake VanderPlas, Senior Data Science Fellow at the University of Washington's eScience Institute gave a keynote speech at SciPy 2015, available on YouTube, on how the interactive tools that scientists and others use in their studies have evolved during the past ten to fifteen years within the IPython community, now evolving into Jupyter.

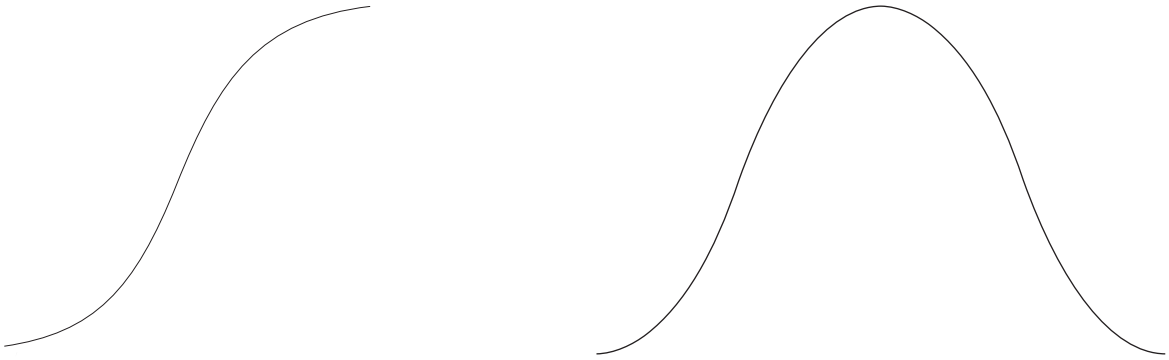
Speaking rather like an information systems architect, Jake VanderPlas said that the modelling and programming tools that scientists use today have evolved within a vast community of toolmakers to produce a coherent structure of tools that fit together as a hierarchical whole, as this diagram illustrates. Yet, no one individual could foresee how this ecosystem, as it is called, could evolve. It has been designed naturally within the community by implicitly following evolution's creative processes, guided by the creative power of Life and the Logos, emanating from the Divine Origin of the Universe.

We see here an example of how we could cocreate a global Sharing Economy, based on the Internet, which would empower as many individuals as possible to realize their fullest potential as human beings following the collapse of the monetary economies of capitalism and communism. Such a scenario is inevitable, for it is false to assume that technological development can drive economic growth indefinitely, one of the most fundamental political assumptions of the world we live in today. With computers increasingly being able to perform tasks more cheaply than humans, the fundamental assumption that humans are both workers and consumers in the economy, articulated in the opening paragraph of Adam Smith's *Wealth of Nations* in 1776, will no longer apply. We humans are the leading edge of evolution, not machines like computers.

We can also see that the global economy is about to self-destruct from the mystical worldview. By the Principle of Unity, all structures in the Universe emerge from nothing and return to nothing at the end of their lifespans. To understand what this means, the diagrams on the next page depict dual patterns that are inherent in all growth and decay processes. The one on the left shows the S-shape of the growth curve, while that on the right, the bell-shape of the life-and-death curve.

In mathematics, the growth curve was first studied in relationship to the growth of the population of Belgium by Pierre François Verhulst, depicting what he called the logistic function. Differentiating this function, to calculate the rate of growth at each point in time, we obtain a bell-shape. This depicts, for instance, the rise and fall of finite fossil fuels, such as coal, oil, and gas, as M. King Hubbert showed in 1956 and 1962. The maximum value of this curve is known as peak oil today, the point in time when the

maximum rate of extraction of petroleum is reached. Conversely, integrating Hubbert's function, giving the area under the curve, brings us back to the growth curve.

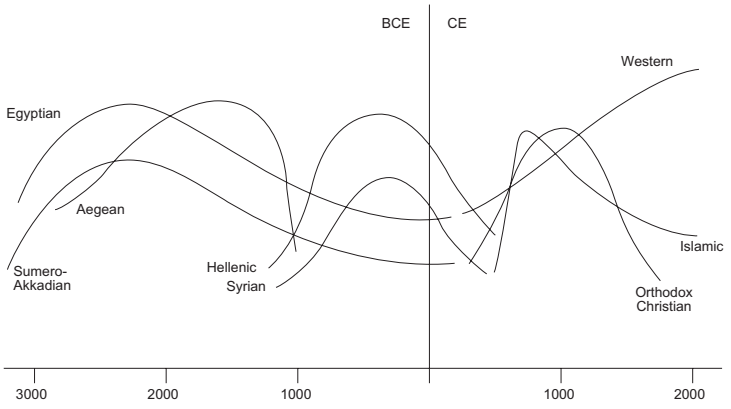


In probability theory, the growth curve is called the cumulative distribution function, which gives the probability that a real-valued random variable X will take a value less than or equal to x. Its dual is called the probability density function or probability mass function if the variable exists only at discrete points, such as the probability of tossing 2 to 12 with a pair of dice. Examples of the former are the normal or Gaussian distribution function, much used in statistics, and the logistic distribution function.

The bell shape of the probability density function also applies to the human population, which is currently moving towards peak population—corresponding to peak oil in Hubbert's curve—when it will begin to decline. When human population reaches its peak will depend on the difference between birth and death rates, with perhaps fertility rates diminishing a little. However, people are now living much longer with improvements in medicine, health and safety, and lifestyle, with maybe half becoming centurions by the end of the century in developed countries, as some forecasts indicate. So how will this affect pensions, with people working for forty-five years supporting a population for the last thirty-five years of their lives? And how will our descendants cope with eleven billion neighbours?

Yet, we already live on an overcrowded, unsustainable planet, with millions living below the poverty line, unable to feed themselves with sufficient nourishment. So if the human population is destined to decrease rapidly, what will happen to our loved ones during this period?

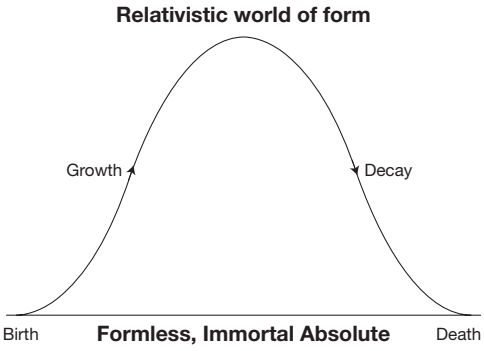
To answer this question, we can use the growth-and-decay curve to illustrate the rise and fall of entire civilizations, putting into graphical form the patterns that Arnold Toynbee discovered by studying the history of the world during the past 6,000 years. He defined about twenty civilizations that have existed during the patriarchal epoch, each passing through essentially the same stages: creative growth, a time of troubles, and a universal state, when the creative energies that originally brought the civilization into being became ossified. In *The Turning Point* in 1982, Fritjof Capra depicted the rise and fall of some of these civilizations around the Mediterranean, displayed here.



Toynbee summarized the reason for the death of civilizations in this way, which quite clearly applies to Western civilization today: "The nature of the breakdowns of civilizations can be summed up in three points: a failure of creative power in the minority [the leaders who brought the civilization into being], an answering withdrawal of mimesis on the part of the majority, and a consequent loss of social unity in the society as a whole."

Western civilization, which still dominates the world through the global economy, is inevitably dying, essentially because the Abrahamic religions, materialistic, mechanistic science, and monetary business teach that we humans are separate from Divinity, Nature, and each other. We can trace this split from Reality to the book of Genesis, where it states that we humans have been created in the image of God, the Impersonal Absolute having been personalized. In contrast, throughout history, mystics have discovered that we are never separate from the Immortal Ground of Being that we all share, especially those in the East, where living in union with the Formless Divine is encouraged rather than discouraged.

The prospects of the imminent death of Western civilization and the global economy are raising much existential fear, which might seem like a problem, whereas it gives us the most wonderful opportunity to realize our fullest potential as a superintelligent, superconscious species before our inevitable demise. We can learn how to grasp this opportunity from Joseph Campbell, another distinguished pattern observer. Viewing our lives as a dynamical system, he called the orbits our journeys in life follow from Alpha to Omega and back again recapitulations of the Cosmogonic Cycle. Abstracting patterns from the myths and fairly tales of many cultures through the ages, he described the journeys that spiritual seekers take in three stages and seventeen steps, culminating when the mystical and mundane are unified in Wholeness.



As Campbell describes in his popular book *The Hero with a Thousand Faces*, the overall purpose of life is to return Home to the Nonmanifest before the death of our bodies. For such a journey in life enables us to live intelligently and consciously in harmony with the fundamental law of the Universe: we are all conceived and born to die, as both individuals and a species. Not to accept this, by holding onto the illusion of a separate self, is to live in denial.

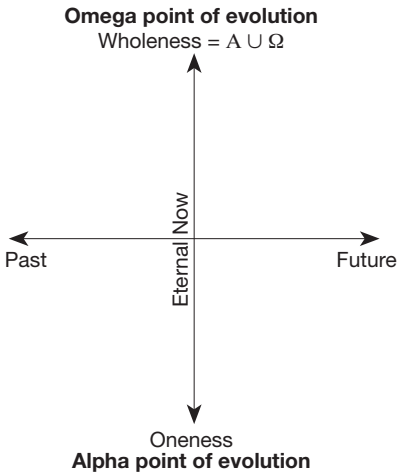
For myself, I have needed to follow this universal spiritual journey in order to deal equanimously with the vision that is revealed by the mathematics of all evolutionary growth processes, described from a sense of Wholeness in this book. At any one point in time, I can see far more than I am able to write down in a well-ordered, articulate manner.

As I have accelerated through evolution's Accumulation Point towards its Glorious Culmination during the past thirty-six years, I have needed to handle a multitude of emotions, at once feeling great joy and gratitude for what has been happening to me and a deep sense of sadness for the ultimate fate of humanity, for those children who will not grow old enough to have children of their own. Given the turbulent state of the world today, it is quite possible that such a fate will befall my own grandchildren and their generation.

This might sound rather pessimistic, when most of us prefer to live in hope, optimistic about the future. But the Cosmic Equation tells us that optimism and pessimism cannot be separated. We need to acknowledge both to live a balanced life, in harmony with the Hidden Harmony. From this balanced perspective, we can learn how to live in the moment, in the Eternal Now, a notion made famous by Eckhart Tolle in *The Power of Now*.

This is the dream. However, as mystics through the ages have discovered, we cannot complete the

spiritual journey in union with the Divine by wanting it. For any such egoic desires get in the way of Self-realization. No technique practiced through time can lead us to the Great Attractor at the end of time. Therein lies one of the many paradoxes that arise from living in a dual world in which opposites are never separate from each other.



For practical purposes, we live in the horizontal dimension of time, remembering the past and visualizing the future. We get up each morning, go about our daily activities, and go back to bed again to sleep at the end of the day. All of us go through this pattern in our daily lives. Yet, in Reality, there is no time. Everything that happens in the Cosmos and hence in our lives takes place in the vertical dimension of time, in the Eternal Now, as this diagram illustrates. So if we are to live in harmony with the fundamental law of the Universe as evolution carries us through its Accumulation Point, we shall find ourselves being turned from the horizontal to the vertical.

We can see why it is essential to live in the vertical dimension of time in order to rise above the level of our machines from this diagram of the basic data-processing operation, which began my researches into the relationship of human and so-called artificial intelligence in 1980.

As this diagram indicates, machines, like computers, operate solely in the horizontal dimension of time. A function in a programming language, like a mathematical function or operator, receives an input, does some processing, and produces an output. Scientists and others interacting with a computer through a Unix shell, for instance, do so through a read-eval-print loop (repl), a term derived from the basic structure of LISP, the first language that attempted to mimic human linear reasoning in a machine.

For instance, a scientist writing a program in Python can enter '2 + 2' into the computer and immediately get the answer '4'. This statement is a snippet of a program, where '+' is a binary function, operating on two inputs '2' and '2'. Here, the shell also acts like a function, processing not only basic data like numbers and words in strings of characters, but also mathematical operators, generalized into functions.

The same mechanistic principle applies to all processes being performed by humans and computers in business. Enterprises receive orders from customers, do some processing, and produce goods and services. But could they do this without human intervention? The answer is an irrefutable 'NO', for humans are not machines and nothing but machines. In Reality, we live in the vertical dimension of time, as the creative power of Life enables us to create structures that have never been seen before, such as this book.

We can see here why there is so much tension in the workplace. On the surface, it acts mechanically. Business functions and processes that take place in organizations are often systematized and documented in a procedure manual, accredited through the ISO 9000 quality management system of the International Organization for Standardization, over a million organizations in 175 countries being so accredited. It is this manual that governs the day-to-day operations of companies and other organizations, which both humans and computer systems are required to follow to ensure the smooth running of the business. So the global economy, as it is organized today, is essentially a gigantic machine, in which we human beings

are mere cogs, such as in a clock, in René Descartes' terms.

The principal reason why many accept being treated as human automata is that money is the primary immortality symbol in the world today. Because we have become detached from Reality—from our Immortal Ground of Being—we have used money, religion, and other cultural structures for this purpose throughout history, as Ernest Becker, the Pulitzer prize-winning author of *The Denial of Death*, points out in *Escape from Evil*. The global economy continues to exist because people have confidence that it will assuage their existential fears, not realizing that capitalism, in particular, is a gigantic confidence trick.

The education system also functions in such a superficial, mechanistic manner. For the most part, we go to school to learn what our teachers want us to learn, which we then regurgitate in examinations. The better we can do this, the higher grades we get and the more successful we are in society's eyes. As my friends tell me, even psychology teachers in universities are reluctant to look beneath the surface to discover what it truly means to be a creative human being.

Herein lies the greatest challenge of our times. We have been acculturated to live in what looks like an alien culture to an increasing number of individuals and groups. How then can we deal equanimously with the practicalities of our lives as evolution passes through its Accumulation Point into chaos? The media is currently publishing many articles showing that the global economy is in a major crisis, on the brink of another major recession, deeper than that in 2008. And the gap between the richest and poorest is getting larger every day, with reports that just a few dozen individuals have more financial wealth than fifty per cent of the global population.

This is an inherently unstable situation. The global economy holds the seeds of its own destruction within it, as I first saw in 1979, when marketing Decision Support Systems for IBM. And, for myself, I receive a tiny pension from working for IBM for twenty years in England and Sweden. So I am dependent for my basic needs for food, clothing, and shelter on an inherently unstable economic system that could collapse at any moment.

These events are taking place when there are many reports in the media of machines taking over our lives. For instance, Martin Rees, a former president of the Royal Society, said in *Our Final Century: Will the Human Race Survive the Twenty-first Century?* "A superintelligent machine could be the last invention that humans need ever make." Similarly, Stephen Hawking, perhaps the most famous scientist in the world, told the BBC on 2nd December 2014, "The development of full artificial intelligence could spell the end of the human race." Such statements grievously deceive the general public, who have placed their trust in science. As most do not have sufficient self-understanding to refute these assertions, they unnecessarily generate existential fear.

If we could realize our fullest potential as a human species, by making the most radical change in the work ethic since our hunter-gatherer forebears settled in villages to cultivate the soil and domesticate animals, the threat of superintelligent computers would disappear.

However, there will still remain a number of other existential risks, as outlined in *The Four Spheres*. Some of these were brought forward to the public on 26th January 2016, when the *Radio Times* published answers to some questions that Stephen Hawking gave at an event in the Royal Albert Hall in London prior to his Reith lectures on black holes. When Paul Ost asked him, "Will the world end naturally, or will man destroy it first?" Stephen Hawking replied:

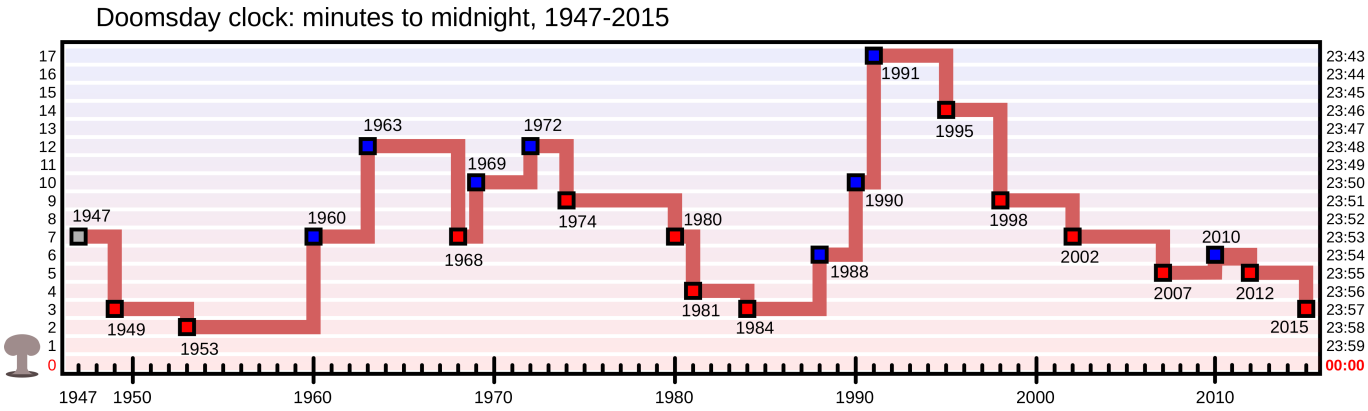
> We face a number of threats: nuclear war, global warming and genetically engineered viruses. Although the chance of a disaster on planet Earth in a given year may be quite low, it adds up over time, becoming a near certainty in the next thousand or ten thousand years. By that time we should have spread out into space, and to other stars, so it would not mean the end of the human race.

However, we will not establish self-sustaining colonies in space for at least the next hundred years, so we have to be very careful in this period. Most of the threats we face come from the progress we've made in science and technology. We are not going to stop making progress, or reverse it, so we must recognise the dangers and control them. I'm an optimist, and I believe we can.

Similarly, Martin Rees has said, "in the twenty-first century, humanity is more at risk than ever before from the misapplication of science. … Special responsibility lies with scientists themselves: they should be mindful of how their work might be applied, and do all they can to alert the wider public to potential perils." As he said, "I think the odds are no better than fifty-fifty that our present civilisation on Earth will survive to the end of the present century without a serious setback."

Another scientist who well understands this is James Lovelock. When Stephen Sackur asked him in a BBC Hardtalk interview in 2010, "What do you think is a viable [population] that Gaia, the planet, can sustain?" said, "I would guess, living the way we do, not more than one billion, probably less". At which Sackur said, "But that's postulating the most dramatic and terrible and unimaginable cull of the human species." To which Lovelock calmly replied, I think it will happen in this century. It will take a miracle for it not to.

Since I was five years old, scientists have also feared that a nuclear war could bring about a premature end to the human race. This scenario is most famously presented as the Doomsday Clock, published each year in *The Bulletin of Atomic Scientists*. As the editor of this year's bulletin wrote, "The Clock has become a universally recognized indicator of the world's vulnerability to catastrophe from nuclear weapons, climate change, and new technologies emerging in other domains." Here is a diagram of how the existential threats to the survival of the human race have looked during my lifetime.



Despite some speculations in the British press before the announcement that the clock would be moved closer to midnight, it was not changed in 2016. As the *Bulletin* said, "The clock ticks now at just three minutes to midnight because international leaders are failing to perform their most important duty—ensuring and preserving the health and vitality of human civilization." But it is important not to blame politicians for the turbulent state of the world today. How can they possibly take responsibility for our lives when they do not understand why they behave as they do?

The central problem here is not so much with democracy as such as with the fact that "government of the people, by the people, for the people", in Abraham's 'immortal' words spoken at Gettysburg on 19th November 1863, is unsustainable and unworkable with today's extremely low level of consciousness. As Alexis de Tocqueville pointed out in the middle of the nineteenth century, democracies are the tyranny of the majority or masses, which John Stuart Mill further explored in *On Liberty*. Rather surprisingly, Ronald Reagan understood the constraints under which democracies and politicians function, for he said in his first inaugural address on 20th January 1981,

> In this present crisis, government is not the solution to our problem; government is the problem. From time to time we've been tempted to believe that society has become too complex to be managed by self-rule, that government by an elite group is superior to government for, by, and of the people. Well, if no one among us is capable of governing himself, then who among us has the capacity to govern someone else? All of us together, in and out of government, must bear the burden. The solutions we seek must be equitable, with no one group singled out to pay a higher price.

What all this means is that the greatest threat to our health, well-being, and survival for as long as possible comes from our deluded, schizoid minds, not from anything outside ourselves. Accordingly, we urgently need radically new social institutions to help guide humanity into the eschatological Age of Light at the end of time, living intelligently and consciously in the Eternal Now. In essence, if we are to reach our fullest potential as a species, we need to collectively transform today's Information, Knowledge, and Wisdom Society into the Mystical Society—the Age of Light—as this diagram illustrates:

Not that this is easy for who we know is generally far more important for people than what we know. As social animals, we prefer to belong to groups of like-minded folk than to venture out beyond the group, moving through the various levels in the spectrum of consciousness. Of central importance here is the family, which is the most fundamental structure in society, as Confucius pointed out in the *Analects*.

However, if children blindly follow what their parents teach them—as the generations have been doing for thousands of years—they will not be able to adapt intelligently to the unprecedented rate of accelerating change we are experiencing today. As Max Planck ruefully remarked in his *Scientific Autobiography*: "a new scientific truth does not triumph by convincing its opponents and making them see the light, but rather because its opponents eventually die, and a new generation grows up that is familiar with it." So it is particularly challenging to cocreate a social environment that is concerned for our entire species, not just some section of it.

We can learn something about how to handle today's challenges from the threat of nuclear devastation that has been hanging over humanity since the Americans dropped the first atomic bombs on Hiroshima and Nagasaki in 1945. In the 1950s, faced with the threat of nuclear war, concerned physicists, such as Hans Bethe and Joseph Rotblat, sought to promote arms control. The latter then gathered eleven distinguished scientists, ten of them Nobel laureates, to sign what became known as the Russell-Einstein Manifesto, issued on 9th July 1955, containing the words, "Remember your humanity, and forget the rest." This Manifesto was followed by the first Pugwash conference on Science and World Affairs in a village in Nova Scotia in July 1957, Rotblat and the Pugwash conference being awarded the Nobel Prize for Peace in 1995.

We urgently need to set up something similar today, for as Martin Rees has said, "In the present century the dilemmas and threats will come from biology and computer science, as well as from physics: in all these fields society will insistingly need latter-day counterparts to Bethe and Rotblat."

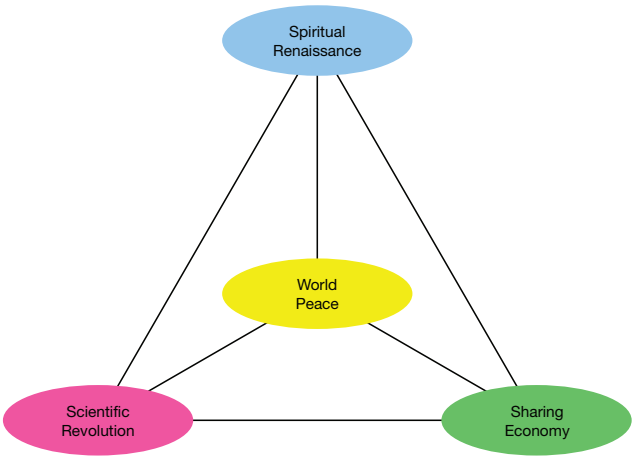
Yes, there are hundreds of institutions in the world addressing these critical issues today. These include the Future of Humanity Institute at Oxford University, directed by Nick Bostrom and funded by James Martin, a fellow alumnus from IBM; the Centre for the Study of Existential Risk at Cambridge University, founded by Martin Rees and Stephen Hawking; the Institute for Noetic Sciences, founded by Edgar Mitchell, following an epiphany he had on his epic journey to and from the Moon; and the Science and Nonduality (SAND) conferences, founded by Zaya and Maurizio Benazzo.

However, there is, as yet, no institution addressing all the critical issues facing humanity today as a coherent whole. Maybe this is because there are so many of them that they look quite overwhelming, when we could suffer from information overload. But when we see that the entire Cosmos has the property of self-similarity, we can see the simplicity underlying the Universe, enabling us to intelligently manage our practical affairs with as much consciousness as we can synergistically muster. If we can send men to the Moon or produce blockbuster movies—two complex operations, involving thousands of people working harmoniously together—surely we are capable of working harmoniously together as a species when our children's future is at stake.

To this end, for the past couple of years, I have been engaged in setting up the Alliance for Mystical Pragmatics with the motto 'Harmonizing evolutionary convergence', endeavouring to fulfil a life-long dream. The overall purpose is to realize this inspiring vision of Teilhard, posthumously made public in 1955: "The way out for the world, the gates of the future, the entry into the superhuman, will not open ahead to some privileged few, or to a single people, elect among all peoples. They will yield only to the thrust of all together in the direction where all can rejoin and complete one another in a spiritual renewal of the Earth."

Another with a similar vision is Thich Nhat Hanh, who said at a Day of Mindfulness in 1993 that the next Buddha—as Maitreya, the 'Loving one'—can only be a community or global sangha, not an individual, as traditionally believed. For Sanskrit *maitreya* means 'friendly, benevolent', from the same Proto-Indo-European base as *community*, from Latin *commūnis* 'shared, common, public', originally in sense 'sharing burdens', from *cum* 'together with' and *mūnus* 'office, duty; gift, present', from *mūnare* 'to give, present'. *Community* is also cognate with Pāli *mettā* 'loving-kindness', the translation of Sanskrit *maitrī*, akin to Buddhist compassion (*karunā*) and love or charity (*agapē*) in Christianity. And when our lives are based on Love, the Divine Essence we all share, we realize that kindness is our True Nature, for *kind* is the native English word for *nature*, the OED tells us, having the same root.

I envisage the Alliance as a network of networking networks, with as many individuals and organizations who wish to join us working harmoniously together with a single purpose: World Peace through the integration of three great global movements, Spiritual Renaissance, Scientific Revolution, and Sharing Economy. These constituents will be engaged in transforming the seven pillars of unwisdom, described in *The Four Spheres*, into the life-enhancing pillars of wisdom.



Such a metamorphosis can only come about through self-inquiry, by following the time-honoured maxim, "Know thyself," which religion, science, and business do not encourage. Yet, there is nothing that could be a source of pride or embarrassment in such self-reflection, as Vimala Thakar pointed out in *Spirituality and Social Action*. As she said, "the inner life or the psychological life is not a private or a personal thing, it's very much a social issue." So if we do not look inwards to discover why we behave as we do, we are actually being antisocial, disturbing the peace and harmony of our beautiful planet Earth.

The Alliance will thus provide a nurturing environment where it is safe to question all the cultural beliefs and assumptions that have traditionally provided people with a precarious sense of security and identity in life. For the only way forward for humanity at the present time is to let go of everything we

have learnt through the entire course of human history. We can no longer afford to blindly pass on what we have learnt from our parents to our children, who will face challenges and opportunities that little in the past can help them resolve.

However, before setting up the Alliance, I feel the need to express the evolutionary vision described in *The Four Spheres* in more mathematical terms. In the mid 1980s, David Bohm told a journalist working for a scientific journal (I forget which) that he did not think that his fellow physicists would accept his theory of the implicate order until he was able to express it in mathematical terms. As he told his biographer F. David Peat in *Science, Order, and Creativity* in 1987, he had known since he was in graduate school at the California Institute of Technology in 1939 that we can only understand physical phenomena from a deeper philosophical ground. Yet, many physicists still believe that mathematical formulations are more significant and that conceptualization just gets in the way.

For myself, the abstract modelling methods that underlie the Internet provide me with the sound coordinating conceptual framework with which to explain why scientists and technologists are driving the pace of evolutionary change at unprecedented rates of exponential acceleration. So, I have not considered it strictly necessary to express this vision in quantitative mathematics. Nevertheless, by doing so now, maybe this could help to complete the final revolution in science, which it seems that it is my destiny to do, for this is the only way that I can interpret my life experiences in a purposeful, meaningful manner.

That is why I have written this essay in the way that I have. Physicists and computer scientists, working within an obsolete paradigm, are perilously deceiving both the democratic masses and the politicians who govern our daily lives. However, I do not write as a mathematician. I am not a polymath, skilled in many specialisms of human endeavour. Rather, I write as a panosopher, a generalist integrating all knowledge into a coherent whole, rather like an information systems architect in business, working with specialists in the various departments of an enterprise. So while the language of Integral Relational Logic has evolved from pure mathematics, it's primal concepts of **Datum**, **data**, **beings**, **patterns**, **sets**, **meaningful structure-forming relationships**, and the universal triple of **class-entity-attribute** lie beneath the foundations of mathematics, just like any other academic discipline.

As such, if I were moved to do so, I could use Integral Relational Logic to produce a coherent map of all branches of mathematics, showing how they all relate to each other as a coherent whole. However, in this essay, I feel that it is more important to focus attention on the mathematics of growth and decay processes, for they help us understand the opportunities that arise from the human predicament a little better. So rather than looking at mathematics as a way of proving theorems or solving particular problems, we see how the mathematical structures that model numerical growth follow a similar developmental pattern, recapitulating the Cosmogonic Cycle.

For mathematical studies are essentially a human activity, not an abstract study devoid of human experience, as Reuben Hersh points out in *What is Mathematics, Really?* And as Yehuda Rav said in *18 Unconventional Essays on the Nature of Mathematics*, edited by Hersh, "Mathematics ought to be viewed as an ever-expanding mansion floating in space, with links constantly growing between previously separate compartments." Accordingly, by going right back to basic principles, if we are to become free of Euclid's notion that mathematics begins with a set of axioms and postulates and then progresses linearly, we need to begin our mathematical reasoning at the Divine Origin of the Universe, corresponding to the basin of attraction in dynamical systems, the initial values of their development towards attractors in mathematics.

Like Leonard Euler's *Introduction to Analysis of the Infinite* from 1748 and Richard Courant and Herbert Robbins' classic textbook *What Is Mathematics?* from 1941, the first section begins with a few basic principles, as seeds. These then grow into structures of the most amazing complexity in a multitude of patterns and relationships, giving great joy when they are perceived.

But it is not necessary to venture far into the complexity that professional mathematicians like to explore. To see the elegant simplicity underlying the Cosmos, the mathematics involved is no more advanced than that which I studied as school and university during the six years either side of 1960, namely algebra and elementary calculus.

I have since little used these mathematical techniques in my business career, so I am distinctly rusty and would welcome corrections to any errors I might have made in the mathematical expressions. I only set out to model mathematics as a coherent whole at the beginning of 2016, when I borrowed a couple of dozen books through the Swedish library system and set out to investigate in more depth than I had previously done the mathematics of dynamical systems.

This is an ongoing learning process, where I am discovering some new mathematical concepts that could help to clarify the exposition, such as generating functions and tetration, as repeated exponentiation, the next hyperoperation after exponentiation itself. This second draft of this essay contains examples of these structures without explaining their underlying generalizing principles, which I intend to do in the third draft.

Regarding interpretations of the mathematics as meaningful pictures, I am aware that some of my interpretations could be different from those of professional mathematicians, not the least because I don't really understand many of the detailed concepts involved in their studies.

Nevertheless, I am quite sure that the overall picture is sound, for all the parts fit harmoniously together as a coherent whole, like a gigantic multidimensional jigsaw puzzle. So I am not attempting to prove anything with deductive logic or linear mathematical proof. Because the dynamical systems that mathematicians study are nonlinear, we need a nonlinear, holographic, fractal-like system of thought to study them, as outlined in this Introduction.

To mirror this Cosmogonic process in a manner that relates directly to experience, this essay begins with an introduction to the 'Malthusian catastrophe', indicating the incompatibilities between arithmetic and geometric series when applied to population growth.

As these series are the most basic ways in which mathematicians model growth, the first section explores these relationships in some detail, without going too far into the complexities of these functions, which I much enjoyed as an undergraduate at the beginning of the 1960s. We begin with the series themselves, before exploring the fundamental characteristics of continuous exponential functions.

However, to study dynamical systems, mathematicians have discovered that they need discrete difference equations, corresponding to the differential equations that they use to model continuous processes. As a gentle introduction to this subject, whose essence is simplicity itself, we next look at how recurrence relations are used in fractal geometry, generating the most beautiful, self-similar patterns.

Finally, in this section, we look at the amazing ways in which numbers can grow in mathematics. First, we look at the ways that finite numbers can grow exponentially, yet remain finite, albeit far beyond our intuitive understanding of such numbers compared to the lifespans of our bodies. These numbers lead into a consideration of the infinity of infinite cardinals and ordinals. These might seem quite mind-

boggling, yet, nevertheless, they are just manifestations of the categorizing mind, leading to the Transfinite Absolute, beyond all categories.

Having looked at the way that the growth of numbers can lead to God, we have the Cosmic Context and Gnostic Foundation that we need to put the growth and decline of human population into proper perspective. We begin with Fibonacci's study of a hypothetical population of breeding rabbits in the thirteenth century before moving on to Verhulst's study of possible population growth in Belgium in the nineteenth. This leads us directly to Robert May's study of a population of fish in the last century, using one of the most fundamental difference equations in chaos theory, known as the logistic map.

The logistic map lies at the heart of the holistic theory of evolution presented in *The Four Spheres*, this essay, and my other writings. But before describing this, in the next section, we look at two 'New-agey' models of evolution: Carl Johann Calleman's study of the exponential nature of the Mayan calendar, and Terence McKenna's fractal-like Timewave Zero. Such models are sometimes dismissed as 'pseudo science' in Wikipedia and elsewhere.

However, they lead directly to two comprehensive models of evolution developed by Nick Hoggard and Ray Kurzweil, both computer scientists. These models are the subjects of the last two subsections in this section. They highlight the split between those who know that humans are the leading edge of all evolutionary processes on Earth and those who believe that technological development can drive economic growth indefinitely.

What is missing from this exposition is the related mathematics of weather systems, for these are rather more complex and I have not studied them in any detail. To focus attention on evolutionary and population growth, this version also does not include some other related mathematical constructs, which are described in my 2014 treatise 'The Theory of Everything'. These include, Claude Shannon's so-called information theory, the Riemann hypothesis, and some further illustrations of the principle of duality in projective and inversive geometry studied by the Japanese from the 1600s to the 1800s, when they were isolated from the rest of the world. 'The Theory of Everything' also contains a couple of simple, but rather technical explanations of why computers can never exceed human intelligence without appeal to the Divine, just using simple reasoning, understandable by the intellect.

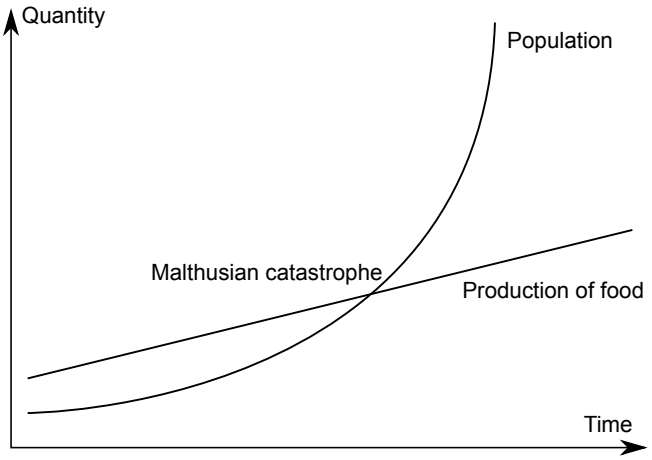
A much fuller explanation of these rather technical issues is available in the *Wholeness* trilogy, which I have not worked on since January 2013. So, at present, I do not know the best way of publishing my thirty-six year study of our rapidly changing times. No doubt, this will be sorted out when a small team is attracted to setting up the Alliance for Mystical Pragmatics.

Finally, I would just like to thank you for reading through this Introduction. It has been far harder to write than the mathematical pages that follow, not the least because I am endeavouring to communicate to people living at all levels in the spectrum of consciousness, from a multitude of different backgrounds. Furthermore, as an autodidact, I am still working in almost complete solitude, with only one friend to occasionally bounce ideas off, seeking the best way of communicating what we all actually know in our hearts, in the depth of being.

This is the second draft of the Introduction, which could well be changed when this essay is turned into book form and when I have found content and language editors to help me publish my writings in the most effective manner. I include some initial thoughts on this in the final section on 'Practicalities'.

Through Evolution's Accumulation Point
Towards Its Glorious Culmination

There are two basic ways in which mathematics depicts growth: additive and multiplicative, which is shorthand for repeated addition. We see this most clearly in arithmetic and geometric progression, a situation that greatly disturbed Thomas Malthus, who wrote in 1798 in *An Essay on the Principle of Population*, "Population, when unchecked, increases only in a geometric ratio. Subsistence increases only in an arithmetic ratio."[1]



Malthus therefore deduced, "The power of population is indefinitely greater than the power in the earth to produce subsistence for man," known today as the 'Malthusian catastrophe', illustrated here. This deduction is based on two fundamental postulates: "First, That food is necessary to the existence of man. Secondly, That the passion between the sexes is necessary and will remain nearly in its present state."[2]

How humanity balances our sexual energies and our need to eat food in the years to come is dependent on many factors, not the least to what extent evolution becomes fully conscious of itself within us. For the more clearly we can see the Big Picture and understand the fundamental laws of the Universe that govern our lives, the more we can act intelligently and consciously in our daily lives. For the most part, we are not aware of these universal laws when we make love to bring children into the world or buy food in supermarkets to feed ourselves. Most have neither the time nor the inclination to pay much attention to the Cosmic information system in which we are all participating as individuals, even though there is enormous interest and wonder in the subject.

During my twenties and thirties, my life followed this conventional pattern. I lived very much in ignorance of what was causing me to behave as I do. The turning point came in the late 1970s, when working in marketing for IBM, when I realized that the global economy holds the seeds of its own destruction within it and that my two children were not being educated to live in the world that would exist when they came to have children of their own. With this insight, in order to take responsibility for my actions, I returned to the central theme of my life, begun as a seven-year-old in 1949. For my deepest passion throughout my life has been to understand what the Universe is, how it is designed, and the root causes of conflict and suffering in the world so that we could all live in love, peace, and harmony with each other.

Starting with myself, then society, and then the Cosmos, most of these studies have avoided mathematics, for I found myself working beneath the foundations of mathematics as this subject is understood today, of only secondary interest to my thesis. However, in January 2016, I felt the need to bring mathematics to the forefront, fully grounded in Integral Relational Logic and integrated into the Unified Relationships Theory. For the pure mathematics of growth and decay processes form a coherent whole, which I find enhances one's understanding of what is happening to us all at the present time when applied to evolution as a whole.

This essay begins by describing the basic mathematical techniques that we need to understand the later sections. We then go on to look at three basic models of the growth in the population of rabbits, humans, and fish, both without and with limits. To set these growth processes into their evolutionary context, we then go on to show why evolution is currently passing through the most momentous turning point in its fourteen billion-year history. Finally, we take a peek at some practicalities, at what needs to happen if we humans are to collectively enter the eschatological Age of Light, living in love, peace, and harmony with each other and our environment at the end of time in the Eternal Now.

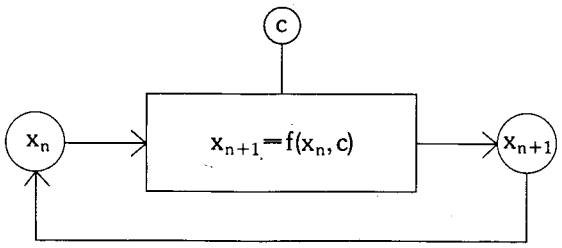
Basic principles

Viewing the Cosmos in terms of structure, form, relationships, and meaning, rather than the matter, space, and time of the physicists and the fire, air, earth, and water of the ancients, we can define evolution in all its forms in this way: *Evolution is an accumulative process of divergence and convergence, proceeding in an accelerating, exponential fashion by synergistically creating wholes that are greater than the sum of the immediately preceding wholes through the new forms and relationships that emerge, apparently out of nothing.*

Mathematics has some difficulty in expressing the growth of structure because the sum of the parts is often greater than the individual parts from the new relationships that are formed. Furthermore, the second law of thermodynamics in physics has great difficulty in accommodating evolutionary growth processes, asserting that the overall trend of the universe is towards a heat death, when all energy will be dissipated in maximum entropy and disorder.

Nevertheless, there are two basic ways of quantifying growth processes in mathematics: as an exponential function and as a recurrence relation. The first of these has the general form q^p, where p and q are a constant, variable, or a combination of these in mathematical expressions. When the exponent p, which denotes power, is an integer, it is often denoted by n, giving us a discrete function, and when it is real, denoted by x, the function is continuous. As a special case of a mathematical expression, the exponent can, itself be an exponential, *ad infinitum*, like a^{b^c}, as Leonard Euler (1707–1783) pointed out when introducing exponentials in his systemic study of infinite series in 1748.[3]

A recurrence relation is often expressed as a difference equation, the discrete form of a differential equation for continuous values. It is essentially a feedback loop, as this diagram illustrates. The difference equation in the box in the middle takes a value x_n to produce x_{n+1}, which is fed back to x_n.[4]



This example of a recursive, iterative function is a first-order one, for only one previous value in the sequence is used. But in general, x_{n+1} could be calculated from two or more previous values, as in the famous Fibonacci series, described on page 31 in Section 'Population growth'. This is an example of a linear recurrence relation. However, to study complex dynamical systems, such as the growth of populations and evolutionary development, $f(x)$ needs to be nonlinear, leading to fractals and chaos theory made famous in the past few decades, although their origins go back much further.

To show how these mathematical structures evolve from reasonably simple ones, this section focuses attention on the basic principles underlying the mathematics of growth. It begins by looking at the

additive and multiplicative characteristics of arithmetic and geometric series before going on to explore the relationship between the exponential function and logarithms. Before looking at the basic mathematics underlying chaos theory in later sections, in this section we then look at a few examples of fractal geometry, pointing in the direction of the multiplicity of patterns in nature.

Finally, in this section, we address some of the greatest difficulties that humans have had over the years in contemplating the huge numbers that arise from exponential growth. These culminate in an infinity of infinite cardinals, which can never be reached, unlike Transfinite Wholeness, which is ever present.

Arithmetic and geometric series

In mathematics, the most basic of all growth processes is the series of integers. For instance, by starting with 1 and successively adding 1, we obtain the arithmetic series 1, 2, 3, 4, 5, 6, 7, and so on. In general, we begin with a and add d each time, giving

$$a_{n+1} = a_n + d$$

where $a_1 = a$.

Alternatively we can begin with 1 and multiple each preceding number by a constant factor, such as 2. In this case, we form the geometric series 1, 2, 4, 8, 16, 32, 62, 128, and so on. In general, if we begin with a and multiply by r each time, we have

$$a_{n+1} = ar^n$$

where $a_0 = a$.

These arithmetical and geometrical series are so fundamental to all number systems that they form the core of Section 2 in Chapter 1 of Richard Courant and Herbert Robbins' classic textbook *What Is Mathematics?*, which Albert Einstein (1879–1955) described as "A bold representation of the fundamental concepts and methods of the whole field of mathematics … Easily understandable."

The two series provide the basis for the two fundamental progressions in mathematics: arithmetical and geometrical, although the etymology of the latter term in this context eludes me. In the former, A_n, the sum of the first n integers, is given by this formula:

$$A_n = 1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

This is quite easy to prove by the mathematical technique of induction, which it is not necessary to explain here. In writing out such an additive sequence of terms, it is often simpler to use the capital Greek letter *sigma*, Σ. So we have:

$$A_n = \sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

In general, beginning with a and adding d each time, gives this formula for the first $(n + 1)$ terms:[5]

$$P_n = \sum_{k=0}^{n} a + kd = \frac{(n+1)(2a+nd)}{2}$$

When $a = 0$ and $d = 1$, this is equivalent to the formula for A_n.

From this formula, we obtain another series of numbers, called the triangular numbers: 1, 3, 6, 10, 15, 21, 28, 36, 45, 55, and so on, for they count the objects that can form an equilateral triangle.[6] This series of numbers is the second diagonal in from the left in Pascal's triangle, illustrated here, named after Blaise Pascal (1623–1662), following the publication of *Traité du Triangle Arithmétique* in 1654,[7] although other mathematicians studied this for centuries before him in India, Greece, Iran, China, Germany, and Italy.[8]

```
          1
        1   1
      1   2   1
    1   3   3   1
  1   4   6   4   1
1   5  10  10   5   1
```

The triangular numbers are thus a subset of the binomial coefficients of the polynomial expansion of $(x + y)^n$, more simply expressed in the binomial theorem as:

$$(x+1)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k}$$

where

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$$

This expression is also known as the number of ways of selecting k items from a group of n items in combination theory, where $n!$ is factorial n, defined as the product of all the integers up to n. So $3!$ is 6 and $4!$ is 24. Here, we have the product of a series of numbers, which grows very fast, denoted by the capital Greek letter *pi*, Π. As a simple example, corresponding to A_n, the sum of the integers, we have:

$$n! = \prod_{k=1}^{n} k$$

Returning to A_n itself, as the triangular numbers are the second diagonal in Pascal's triangle, we have

$$A_n = \frac{n(n+1)}{2} = \binom{n+1}{2}$$

These mathematical constructs are special cases of more general constructs in two ways in the interconnected forest of trees in mathematical structures. First, the binomial theorem is a special case of the multinomial theorem, giving the multinomial coefficients for $(x_1 + x_2 + \ldots + x_m)^n$:[9]

$$(x_1 + x_2 + \cdots + x_k)^n = \sum_{k_1+k_2+\cdots+k_m=n} \binom{n}{k_1, k_2, \ldots k_m} \prod_{1 \le t \le m} x_t^{k_t}$$

where

$$\binom{n}{k_1, k_2, \ldots k_m} = \frac{n!}{k_1!\,k_2!\,\ldots k_m!}$$

Although this formula looks rather complex, the interesting point about it from a structural perspective is that in the trinomial, Pascal's triangle becomes Pascal's pyramid or tetrahedron, with each level in the pyramid being a triangle, corresponding to the lines in each level of Pascal's triangle. For instance, the fifth level in the trinomial is given by this triangle, inverted so as not to confuse it with Pascal's triangle itself.[10]

1 5 10 10 5 1
5 20 30 20 5
10 30 30 10
10 20 10
5 5
1

The quadrinomial is not so easy to visualize, as I discovered when I looked at its first few levels in January 2016. For each level is a tetrahedron of coefficients within a four-dimensional simplex. This process continues indefinitely for the number of components in n-dimensional simplexes (vertices, edges, faces, etc.) display the same combinatorial patterns of growth as multinomial coefficients, as H. S. M. Coxeter (1907–2003), one of the greatest geometers in the twentieth century, tells us in *Regular Polytopes*.[11]

The point of mentioning this example is that it illustrates the way that apparently different branches of mathematics share the same underlying patterns, as with all of mathematics and, indeed, with all disciplines of human knowledge, as we see in the abstract modelling methods underlying the Internet. Furthermore, multinomials and polytopes are examples of how mathematics, as a discipline, grows in complexity from a few simple constructs. As this is a universal principle, mathematicians make much use of it in their studies of growth processes, as we see in this essay.

As another example of generalization, the sum of the integers is a special case of the polynomial expansion of the general power series:

$$\sum_{k=1}^{n} k^m$$

When *m* is a positive integer, this leads to another fascinating sequence of numbers, called Bernoulli numbers, after Jaques/Jakob/James Bernoulli (1654/55–1705), as the coefficients of the polynomial expansions. These numbers are of such central importance, that calculating them was the purpose of the very first computer program ever published in 1843 by Ada Lovelace (1815–1852), included at the end of her memoir to Menabrea's 'Sketch of the Analytical Engine'.[12] As these Bernouilli numbers are more complex than multinomial coefficients, not surprisingly, she did not do so without considerable effort, saying in a letter to Charles Babbage (1791–1871), "I am in much dismay at having got into so amazing a quagmire & botheration with these *Numbers*."[13]

If *m* is negative in the power series, we are led to one of the most important unsolved problems in mathematics, known as the Riemann Hypothesis, after Bernhard Riemann (1826–1866), who posited it in 1859. In 2014, I wrote a summary of this hypothesis—on the way it relates to prime numbers and quantum physics—following a broadcast on Swedish television on this rather esoteric branch of mathematics. You can find my outline in my treatise *The Theory of Everything: Unifying Polarizing Opposites in Nondual Wholeness*.[14] For, as Einstein wrote in an essay on scientific method in 1936, no matter how complex mathematics might become, "The whole of science is nothing more than a refinement of everyday thinking."[15]

Turning now to geometric progression, if we begin with a number *a* and successively multiply each term by *r*, we obtain G_n, given by this formula:

$$G_n = \sum_{k=1}^{n} ar^k = a\frac{1-r^{n+1}}{1-r}$$

When $r > 1$, the geometric series clearly diverges to infinity. However, when $r < 1$, even an infinite number of terms converges to a finite number, giving:

$$\sum_{k=1}^{\infty} ar^k = \frac{a}{1-r} = \frac{ad}{d-1}$$

where $d = 1/r$, denoting that we divide each term by a number greater than 1. For instance, if $d = 2$ and $a = 1$, then

$$1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots = 2$$

With different values for *d* and *a*, this formula plays a central role in fractional-reserve banking and in calculating when evolution passes through the most momentous turning point in its fourteen billion-year history, called its Accumulation Point in systems theory, or more familiarly its Singularity Point, the central theme of this essay. We look at this vitally important issue on page 62 in Subsection 'Evolution as a dynamical system'.

The exponential function and logarithms

A classic example of a geometric series is that for compound interest, which constantly accumulates, year by year. For instance, if we invest $1000 dollars at 5% per annum, then $a = 1000$ and $r = 0.05$ in the formula

for G_n, giving the way that the principal increases over time. But what happens if the interest is paid more frequently than once a year? This is something that Bernoulli considered in 1683.[16]

To keep things as simple as possible, if the initial premium is 1 and the interest is 100%, then at the end of a year, the premium doubles. Now suppose that interest is paid at 50%, half yearly. Then, after six months, the principal is 1.5 and after a year, it is $1.5^2 = 2.25$. In general, if interest is paid at n intervals during the year, then the principal at the end of the year is

$$\left(1+\frac{1}{n}\right)^n$$

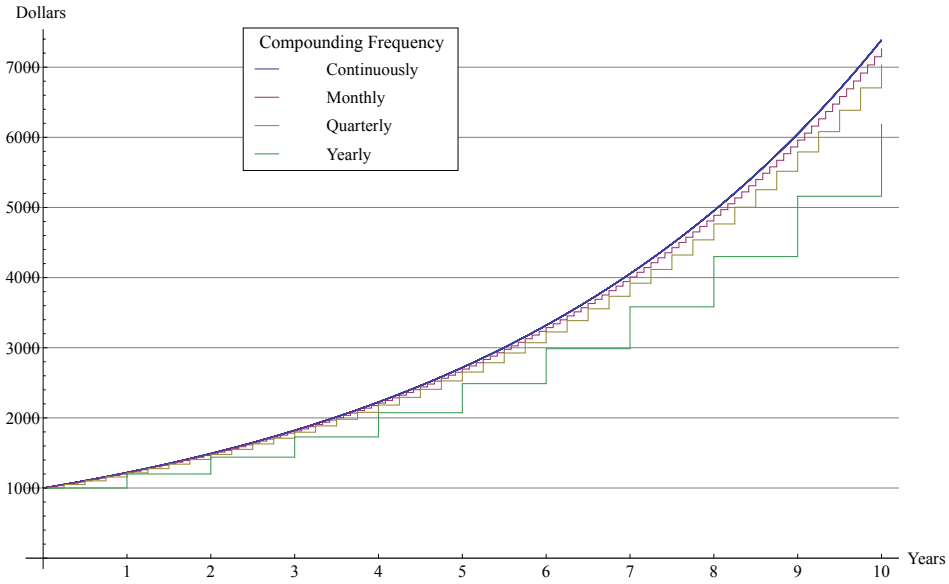
Now, Bernoulli noticed that as n got larger and larger, this number converged to a limit, which we know today as e, often called Euler's number. So we have:

$$e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$$

where e is approximately 2.71828, of equal significance to π (about 3.14159), known to the Greeks as the ratio of the length of the circumference of a circle to its diameter. More generally, if the annual interest is r compounded continuously, the principal at time t is given by this formula:

$$P(t) = e^{rt}$$

Here is a diagram of how compound interest tends to the exponential function with increasing compounding frequencies.[17]



Now, while there are no patterns in the sequence of decimal digits in e, Euler discovered many patterns in this fundamental mathematical constant.[18] For instance, we can expand the formula $(1 + x/n)^n$ for e^x using the binomial theorem, giving

$$e^x = \sum_{n=0}^{\infty}\frac{x^k}{n!} = 1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots$$

And, for $x = 1$,

$$e = \sum_{n=0}^{\infty}\frac{1}{n!} = 1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\cdots$$

Now, this formula for e^x leads to a fascinating relationship with the trigonometric functions.[19] For

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$$

and

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots$$

If we now set $x = i\theta$, where $i = \sqrt{-1}$, we have perhaps the most elegant of all the deep structures in mathematics, called Euler's formula, which shows the relationship between analysis and trigonometry:[20]

$$e^{i\theta} = \cos\theta + i\sin\theta$$

Replacing θ by π or π/2 gives the most amazing formula in mathematics, one that expresses a relationship between five fundamental constants in the simplest possible terms, known as Euler's identity:

$$e^{i\pi} = -1 \quad \text{or} \quad e^{i\pi} + 1 = 0$$

Let us now turn our attention to the relationship between arithmetic and geometric series, illustrated in this table:

0	1	2	3	4	5	6	7	8
1	2	4	8	16	32	64	128	256

The German monk Michael Stifel (1487–1567) published such a table in *Arithmetica Integra* in 1544, pointing out that the sum of two terms in the upper arithmetic series has a connection with the corresponding product of two terms in the lower geometric series.[21] For instance,

$$8\times32 = 2^3\times2^5 = 2^{3+5} = 2^8 = 256$$

This relationship probably gave John Napier (1550–1617), Baron Merchiston of Scotland, the idea of developing a procedure that would substitute the operations of addition and subtraction for those of multiplication and division, making calculations very much easier. Accordingly, Napier set out to pair the terms of a geometric series with those of an arithmetic one, spending twenty years calculating what he was to call logarithms 'reckoning number', from Greek *logos* 'reckoning, ratio' and *arithmos* 'number'. He published his results in 1614 in a small Latin volume of 147 pages—90 of which were tables—with the title *Mirifici Logarithmorum Canonis Descriptio* (*A Description of the Marvellous Rule of Logarithms*). His son Robert then posthumously published Napier's account of how the tables were constructed, written earlier, in *Mirifici Logarithmorum Canonis Constructio* (*The Construction of the Wonderful Canon of Logarithms*).[22]

There is no need to go into Napier's method of calculating logarithms in detail because it doesn't enhance conceptual understanding, not the least because he had no idea of the concept of base for logarithms, as we know it today. We only need to note that Napier used a formula not unlike this, effectively using a base of 1/*e* for his calculations.[23]

$$\frac{1}{e} = \lim_{n\to\infty}\left(1 - \frac{1}{n}\right)^n$$

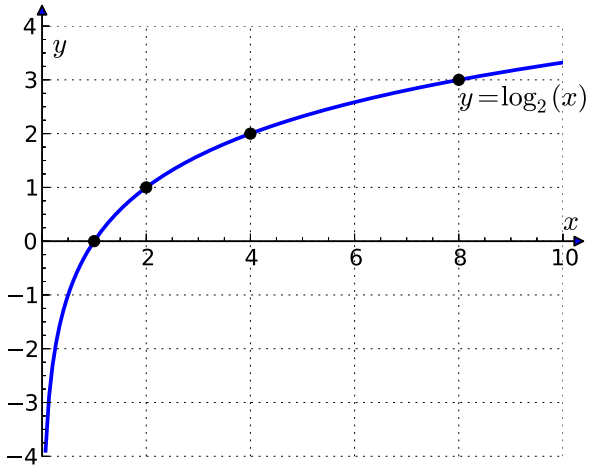
Napier's concept of logarithms quickly caught on, not the least with Johannes Kepler (1571–1630), who had performed thousands of tedious calculations without the use of logarithms in discovering the first two laws of planetary motion, published in *New Astronomy* in 1609, ignored by Galileo but not Newton. In 1620, Kepler wrote a laudatory oration to the Baron of Merchiston, not knowing that he had died,[24] and then, in his own industrious manner, seeking to go to the heart of the matter, he set out in the winter of 1621–22 to write his own book on logarithms, published in 1623. Kepler was thus able to complete the task for which he had been appointed as *Imperial Mathematicus* to the Holy Roman Emperor in Prague in 1601, following the death of Tycho Brahe: to publish Tycho's one thousand measurements of the stars and

those of planetary motion. Following a dispute with Tycho's relatives, these were eventually published in 1627 as *Tabulæ Rudolphiæ* 'Rudolphine Tables', in honour of Rudolf II.[25]

In the meantime, in England, Henry Briggs (1561–1631) embarked on the tedious task of preparing the first set of common, or Briggsian, logarithms using 10 as a base, published in 1624 as *Arithmetical Logarithmica*, which the Dutchman Adrian Vlacq (1600–1666) expanded in a second edition in 1628, calculated to 14 and 10 decimal places. Also, in 1622, William Oughtred (1574–1660) invented the slide rule, as a mechanical device based on the additive power of logarithms.[26] Thus were established the basic tools I needed as a mathematician at school and university in the 1950s and 60s, not now much used following the invention of the pocket calculator and personal computer.

However, while logarithms were of great practical use, it was to take some time before they were fully understood. Today, it is obvious to us that logarithms are the dual of exponents. For, if $x = b^t$, then we know that $t = \log_b x$. That is, t is log to the base b of x. But it wasn't until the 1680s that this relationship began to appear in the consciousness of mathematicians and hence in their writings. And it was not until 1748, when Euler published *Introductio in analysin infinitorum* (*Introduction to the Analysis of the Infinite*), laying down the foundations of modern mathematical analysis, that a full understanding was developed.[27]

There is no need to go into this long learning process, one of many examples illustrating the way evolution progresses in the noosphere. All that is relevant for this essay is to highlight a couple of points that could help us become more aware of what is happening to humanity at the present time.



First, this diagram plots the logarithmic function for base 2, corresponding to Stifel's table on page 7, denoted by the bullets on the curve. For instance, we can see that log 1 is 0, which is true of all logarithms, no matter what their base. Also, $\log_2 2 = 1$, which is an example of the general principle $\log_b b = 1$. The graph then shows how the next few terms on the x-axis, which are in an arithmetic series, become a logarithmic scale on the y-axis.

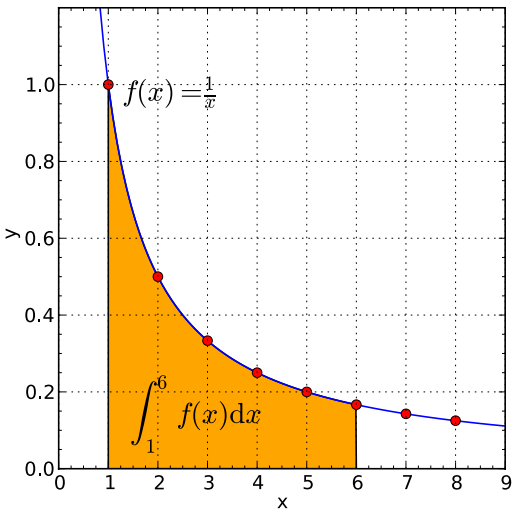
This is particularly useful when plotting geometric and exponential phenomena, for their graphs rapidly disappear off the page using arithmetic scales. On the other hand, when using a semilogarithmic chart, in which exponents follow a geometric progression, exponential growth can be depicted as a straight line, nevertheless still stretching out to infinity. Or, in the case of a diminishing geometric series, when $r < 1$ in the expression for G_n on page 5, the straight line crosses the x-axis, when $y = 0$, at a finite, limiting point. As evolution as a whole can be represented as a diminishing geometric series, such a plot is especially useful in seeing humanity's place in the overall scheme of things, depicted on page 63.

We now need to ask what is so special about logarithms to the base e, which is approximately 2.71828. Surely it would be much simpler to use an integer as a base. For instance, if we express the x-axis in the plot of $\log_2 x$ in binary, then 2, 4, and 8 become 10, 100, 1000, as they are in $\log_{10} x$, or indeed any other number system such as base 3, 7, or 16, with hexadecimal digits used in computers. So why did Nicholas Mercator (c. 1619–1687) call logarithms to the base e 'natural logarithms' in *Logarithmotechnia* in 1668? What is natural about e?

As an aside, Mercator presented the following relationship in his book, which Isaac Newton (1642/43–1727) had already discovered, but not published before being appointed as the Lucasian professor of

mathematics at Cambridge University, on the recommendation of Isaac Barrow (1630–1677), the then holder of this prestigious chair.[28]

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \cdots$$



Maybe this elegant expansion, one of many in mathematics, gives us a clue to the naturalness of the number *e*. But, for me, to see this clearly, we need a little calculus. Gottfried Wilhelm Leibniz (1646–1716), I think it was, showed that log *x*, dropping explicit base *e*, is the integral of the hyperbola 1/*x*, where he and Newton defined the integral as the area under a curve. Here is the mathematical expression with an example diagram. The area shaded orange is log 6 to base *e*. In general,

$$\log a = \int_1^a \frac{1}{x} dx$$

Now Leibniz and Newton also showed that the slope of a curve at a particular point, obtained through differentiation, is the inverse of integration, which is the fundamental theorem of the calculus, not immediately intuitively obvious.[29] So if we differentiate log *x*, we should get back to 1/*x*. But what is the differential of $\log_b x$? Well, to convert logs from one base to another, we use this formula:

$$\log_b x = \frac{\log_k x}{\log_k b}$$

So, differentiating this function, using ln as $\log_e$, as is often done in mathematics, we have, using Leibniz's notation for the differential:

$$\frac{d}{dx}\log_b x = \frac{d}{dx}\left(\frac{\ln x}{\ln b}\right) = \frac{1}{\ln b}\cdot\frac{d}{dx}(\ln x) = \frac{1}{x\ln b}$$

Now when *x* = 1, the slope of the logarithm function as it crosses the *x*-axis is 1/ln(*b*). This number is greater or less than one depending on whether *b* is less or greater than *e*. For instance, ln 2 and ln 10 are 0.693147 and 2.302585, respectively. But when *b* = *e*, the slope at this critical point is 1. Wikipedia suggests that this key characteristic of *e* is what makes logarithms to this fundamental mathematical constant natural,[30] Euler coming to a similar insight through the use of infinite series, without calculus.[31] This explanation is rather elusive, not nearly as obvious as the concept of π, to which *e* is closely related.

As another aside, even though there is no pattern in the decimal digits of π, Leibniz made another beautiful mathematical discovery,[32] which Euler further developed with many other infinite series and products in terms of the prime numbers for functions of π:[33]

$$\frac{\pi}{4} = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \cdots$$

There is just one more significant point to note. The differential of a function with respect to time indicates how fast it is changing. For instance, if the function is a constant, such as *y* = *c*, then

$$\frac{dy}{dt} = 0$$

And if *y* = *ct*, that *y* is changing steadily in a straight line, then

$$\frac{dy}{dt} = c$$

And if $y = ct^2$, indicating the y is a parabola, then

$$\frac{dy}{dt} = 2ct$$

If we now differentiate y a second time, we obtain an expression for acceleration, which is constant in this case:

$$\frac{d^2y}{dt^2} = 2c$$

In general, if $y = ct^n$, then

$$\frac{d^ny}{dt^n} = cn!$$

In other words, eventually, after a finite number of differentials, the result is a constant. But this is not the case for $y = e^t$. For

$$\frac{dy}{dt} = e^t$$

The differential of the exponential function is the exponential function itself. In other words, the exponential function, as an expression of accumulative processes, such as evolution, never slows down if there are no physical constraints for it do so. The rate at which acceleration changes and accelerates is also exponential, and so on ad infinitum!

This subsection has thereby described one aspect of the basic mathematics we need to understand the unprecedented rate of evolutionary change we are experiencing today, as scientists and technologists, aided and abetted by computer technology, drive the pace of scientific discovery and technological invention at ever-increasing, exponential rates of acceleration.

Fractal geometry

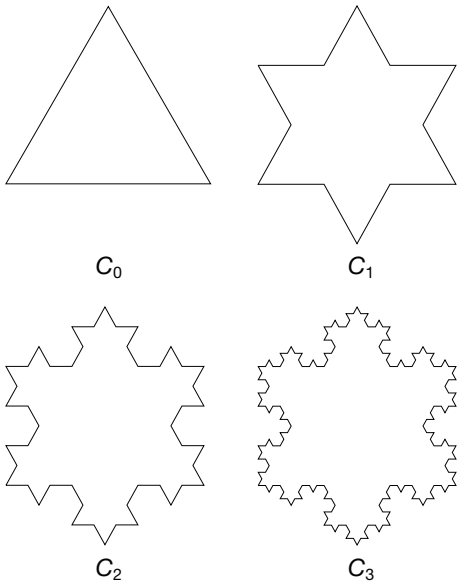
We now need to turn to recurrence relations, the discrete versions of continuous exponential functions. As an introduction, I feel that it is helpful to look at the basics of fractal geometry, which preceded the coinage of the term *fractal* by Benoit B. Mandelbrot (1924–2010) in 1975, like me, an employee of IBM for a time. As he says, *fractal* derives from Latin *frāctus*, past participle of *frangere* 'to break, create irregular patterns', also root of *fragment*, *fracture*, and *fraction*. Interestingly, Mandelbrot points out that *fractal* is the etymological opposite of *algebra*, which derives from Arabic *al-jabr* 'the reunion of broken parts, bone-setting, the surgical treatment of fractures', from *jabara* 'reunite, restore'.[34] The mathematical sense comes from the title of a book *ilm al-jabr wa'l-muqābala* 'the science of restoring what is missing and equating like with like', by the mathematician al-Ḵwārizmī around 820.[35]

This is similar to the etymological opposites of *science* and *art*, which have Proto-Indo-European (PIE) bases **skei-* 'to cut, split' and **ar-* 'to fit together', respectively. These are also the roots of *schizophrenia* 'split mind' and *coordinate*, *reason*, *harmony*, *order*, and *arithmetic*, respectively. So art is a synthesizing activity, healing the split mind, putting back together what analytical science has divided, from Latin *scīre* 'to know', meaning 'to separate one thing from another, to discern'.

A central notion of fractals is that of self-similarity, which applies even more to regular patterns than irregular ones. Let us look at one of simplest of these, as Mandelbrot did in introducing the subject in *The Fractal Geometry of Nature*.

Like him, we begin with the Koch curve, three of which form the Koch snowflake, arranged in a triangle. This was discovered by Helge von Koch (1870–1924), a Swedish mathematician, in 1904,[36] illustrated on the next page. Like so many growth processes, the snowflake begins with a seed or initiator,

which then grows in a regular pattern. In the case of the snowflake, the initiator is an equilateral triangle (C_0) and the iterative generator is a triangle pasted on each of the sides, ⅓ the length of the side of the starting triangle, giving C_1. C_2 and C_3 show the next two iterations, a process that can be continued indefinitely, with some rather strange results.



First, the snowflake is a continuous curve that does not have a tangent at any point. Koch wasn't the first to find such a function, as he acknowledged in his 1904 paper. In 1872, Karl Weierstrass (1815–1897) presented a paper at the Royal Prussian Academy of Sciences, which proved that this function is not differentiable at any point for certain values of the constants *a* and *b*:

$$f(x) = \sum_{n=0}^{\infty} b^n \cos(a^m x\pi)$$

In this paper, first published in 1875, Weierstrass mentions that Bernhard Riemann (1826–1866) had found this continuous function to be nondifferential in 1861, perhaps earlier:

$$g(x) = \sum_{n=0}^{\infty} \frac{\sin(n^2 x)}{n^2}$$

But Riemann's proof was never published, so it was left to Weierstrass to shatter the belief systems of mathematicians at the time.[37] Charles Hermite (1822–1901) labelled such functions as $f(x)$ and $g(x)$ 'monsters',[38] as, indeed, they appear to the casual observer, like me. They were also categorized as counter-intuitive, 'pathological', or even 'psychopathic', as Mandelbrot points out,[39] and were largely ignored by the contemporary mathematical community.

Another fascinating feature of the Koch snowflake is that its perimeter has an infinite length, yet it encloses a finite area. We can see this quite easily using the geometric function on page 5. First, at each iteration, the number of segments increases by four and the length of each segment is reduced by a third. So the perimeter increases by a factor of 4/3 at each iteration, *r* in a geometric progression. So if the original length of side of the triangle is *s*, after the *n*th iteration, the length of perimeter of C_n is $3s(4/3)^n$, which tends to infinity as *n* tends to infinity.

However, the area contained in the snowflake remains finite. For the number of triangles t_n added in the *n*th iteration is:

$$t_n = 3 \cdot 4^{n-1} = \frac{3}{4} \cdot 4^n$$

And the area of each added triangle is one ninth of the triangle added in the previous iteration, giving, where a_0 is the area of the original triangle:

$$a_n = \frac{a_{n-1}}{9} = \frac{a_0}{9^n}$$

So the total new area b_n added in the *n*th iteration is

$$b_n = t_n \cdot a_n = \frac{3}{4} \cdot \left(\frac{4}{9}\right)^n \cdot a_0$$

The total area of the snowflake after *n* iterations is thus:

$$A_n = a_0 + \sum_{k=1}^{n} b_k = a_0\left(1 + \sum_{k=1}^{n}\left(\frac{4}{9}\right)^k\right)$$

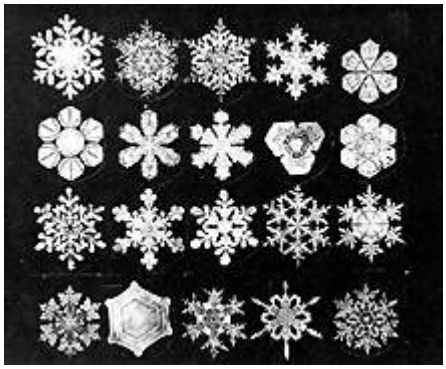
Using the formula for G_n on page 5 to express the sum gives

$$A_n = a_0\left(1 + \frac{3}{5}\left(1 - \left(\frac{4}{9}\right)^n\right)\right) = \frac{a_0}{5}\left(8 - 3\left(\frac{4}{9}\right)^n\right)$$

Now the *n*th power of 4/9 tends to zero as *n* tends to infinity. So the limiting sum of the area is

$$A_\infty = \frac{8a_0}{5} = \frac{2\sqrt{3}s^2}{5}$$

In other words, while the perimeter of the Koch snowflake expands to infinity, the area increases by just 60%.



Of course, the Koch snowflake is so called because like so many fractal structures it is similar to forms in nature, both following the fundamental laws underlying all growth processes in the Cosmos. Amazingly, Wilson 'Snowflake' Bentley (1865–1931) was a farmer in Vermont who took some 5,000 photographs of snowflakes against black velvet, some of which are depicted here from his official website. As he said, "Under the microscope, I found that snowflakes were miracles of beauty; and it seemed a shame that this beauty should not be seen and appreciated by others. Every crystal was a masterpiece of design and no one design was ever repeated. When a snowflake melted, that design was forever lost. Just that much beauty was gone, without leaving any record behind."[40]

But back to fractals, which have another curious feature. Fractal dimensions are not necessarily integers, as they are for lines, squares, and cubes, for instance, with 1, 2, and 3 dimensions, extended into hypercubes or regular orthotopes with any number of dimensions. As Coxeter points out, these are not related to Hermann Minkowski's four space-time dimensions,[41] or the various number of dimensions being postulated by string theorists for physical reality.

It is not easy to intuitively visualize noninteger dimensions any more than it is easy to see structures in four or more dimensions. These are essentially mathematical constructs, which nevertheless deserve a meaningful explanation, for they lie at the core of Mandelbrot's *The Fractal Geometry of Nature*. To give meaning to the concept of fractal dimension, he wrote an article in 1967 titled 'How Long Is the Coast of Britain? Statistical Self-Similarity and Fractional Dimension',[42] much inspired by a rather obscure paper that "chance (or fate) [had] put my way"[43] by Lewis Fry Richardson (1881–1953) titled 'The Problem of Contiguity: An Appendix to *Statistics of Deadly Quarrels*', 1961,[44] the book having been published the year earlier.

Before we look at Richardson's contribution to fractals, I should mention that he is best known for his book *Weather Prediction by Numerical Analysis*, first published in 1922, in which he developed the basic algorithms used in modern weather forecasting based on data collection points at regular intervals covering the globe. He proposed a square grid of 200 km, giving 3,200 cells. From this he estimated "64,000 computers would be needed to race the weather for the whole globe."[45] It was a race against time, for the weather would have changed before all the calculations could be made.

However, as Sydney Chapman pointed out in his 1965 introduction to this seminal book, Richardson had made an uncharacteristic error in his calculations.[46] Given that the radius of the Earth is about 6,371

kms and that the area of the surface of a sphere is $A = 4\pi r^2$, he would have needed 13,155 squares to cover the Earth. The number of human computers was thus a quarter of what it should have been. It seems that Richardson used the formula for the area of a circle rather than that for the surface of a sphere. I well know this problem, for I too make elementary errors in calculations, which is why I have never been very good at accounting or computer programming.

However, as Peter Lynch pointed out in his 2007 foreword to the republication of Richardson's book, even if the human calculators had been available, the partial differential equations did not take into consideration the delicate dynamical balance between the fields of mass and motion prevailing in the atmosphere. So any 'forecast' would have been a failure, being contaminated by spurious noise, a problem that Richardson later understood. Nevertheless, "While this book had little effect in the short term, his methods are at the core of atmospheric simulation and it may be reasonably claimed that his work is the basis of modern weather and climate forecasting."[47]

Indeed, Edward N. Lorenz acknowledged Richardson's pioneering efforts in a paper dated February 1963 titled 'The Predictability of Hydrodynamic Flow', published a month after 'Deterministic Non-periodic Flow', which led Tien-Yien Li and James A. Yorke to apply the word 'chaos' to such phenomena in 1975, as mentioned on page 42. So today, with an increasing understanding of the dynamics of weather systems, and the availability of satellite data and supercomputers doing the zillions of calculations, weather forecasting has been improving over the years. Nevertheless, there is still much to learn about the psychological influence of human behaviour on climate change.

But back to Richardson's influence on the development of fractals. As a Quaker and pacifist during the First World War, his other major of field of interest was to use mathematics and psychology to find the root cause of wars, not unlike my own lifetime passion, having been born near London in the middle of the Second World War. Being much interested in the mathematical psychology of warfare, Richardson decided to search for a relation between the probability of two countries going to war and the length of their common frontier.[48]

However, while collecting data, he found that there was considerable variation in the various published lengths of international borders between seven different countries. For example, that between Spain and Portugal was variously quoted as 987 or 1214 km, and that between the Netherlands and Belgium as 380 or 449 km. To discover why this was so, Richardson used equal-sided polygons of increasingly short sides to measure the length of various land frontiers and seacoasts. He did this with a pair of dividers on maps with suitable scales, moving from point to point in both directions, sometimes giving slightly different results. The length of the border was the length of the polygon side times the number of segments.[49]

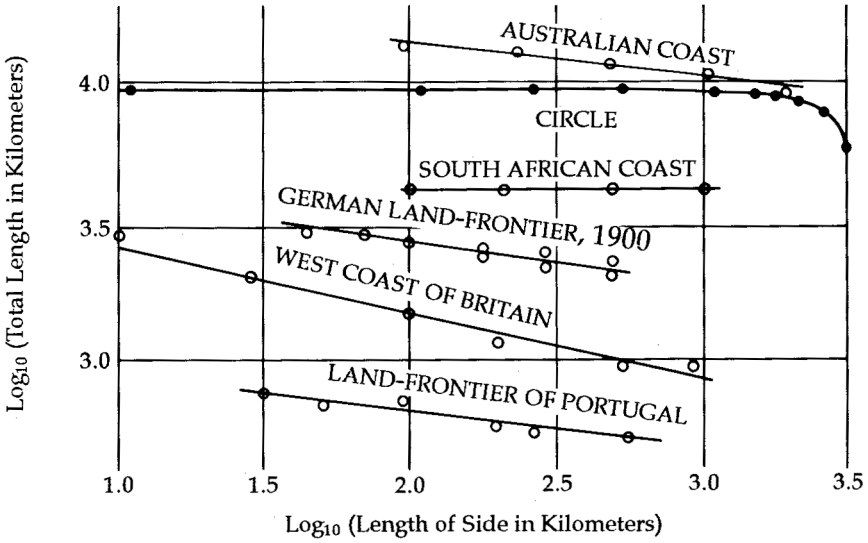
In measuring the lengths of borders in this way, he discovered these relationships between the length of the side of the polygon (s) and the length of the border ($L(s)$):

$$L(s) = cs^a \quad \text{or} \quad \log L(s) = b + a \log s$$

In other words, on a double logarithmic scale, there is a straight-line relationship between the two variables, depicted in the next diagram, reproduced in Mandelbrot's seminal paper 'How Long is the Coast of Britain?'.

I have found these figures for a and b on the Internet,[50] which are slightly different from the values for a in Richardson's 'Appendix to *Statistics of Deadly Quarrels*'. The greater the slope of the line the 'rougher' the border being measured. However, rather than using a to measure this roughness, Mandelbrot used D for dimension, where: $D = 1 - a$.

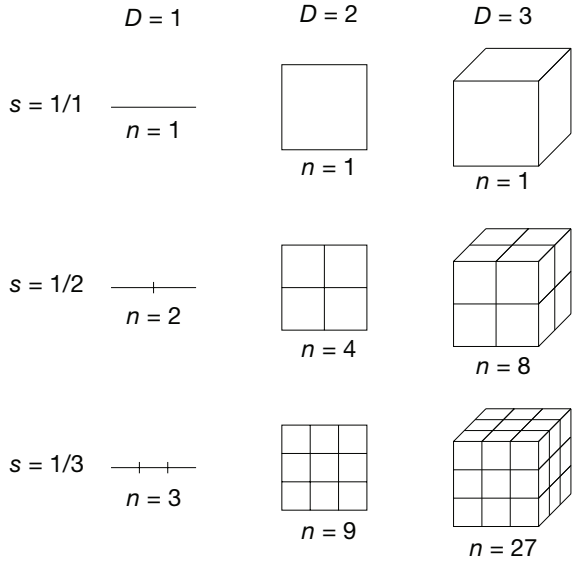
Country	b	a	D
Australia	4.4	-0.13	1.13
South Africa	3.8	-0.04	1.04
Germany	3.7	-0.12	1.12
England	3.7	-0.24	1.24
Portugal	3.1	-0.12	1.12



This is similar to the formula of fractional dimension that Felix Hausdorff (1868–1942) devised in 1919 when studying the rather obscure mathematical notion of the measure of a set (defined on page 24), not directly related to geometry.[51] However, neither Richardson nor Mandelbrot mentions Hausdorff in their pieces. The latter later made the connection in *The Fractal Geometry of Nature*, writing that fractional dimension is more appropriately called 'fractal dimension'.[52]

Regarding the coastline of Great Britain, Mandelbrot, following Richardson, realized that how long it is depends on how long the ruler is that is being used for the measurement, known as the 'coastline paradox', the idea that a landmass does not have a well-defined perimeter.[53] For instance, this diagram from Wikipedia measures the British coastline with measuring rods of 200, 100, and 50 km in length, giving the length of the coastline as about 2350, 2775, and 3425 km, respectively. The shorter the scale, the longer the measured length of the coast. As mathematics is not inhibited by physical constraints, the effective length of the coastline of Great Britain or of any other self-similar, fractal-like curve is infinite, like the Koch curve.

In the case of the Koch snowflake, the fact that it has an infinite length indicates that in some sense it has more than one dimension, but less than two, as it does not fill the Euclidean plane, not on its own, anyway. But how is this dimension to be determined? Well, this hinges on the notion of scaling, which also plays a part in chaos theory. What we are endeavouring to do here is find a function that expresses dimension D in terms of scaling factor s and n, a number that denotes the level of detail revealed as a structure is scaled, such as the number of segments in the Koch snowflake.



To develop such a formula, mathematicians first find a formula that expresses n in terms of two known variables, D and s, illustrated in this diagram. From these pictures, we

obtain:

$$n = \frac{1}{s^D}$$

If we now take logs of both sides, we have

$$D = \log_{1/s} n = \frac{\log n}{\log 1/s}$$

where the logs are to any base because the ratio of two logs is independent of the base, as Euler proved. In the case of the Koch snowflake, at each iteration, the number of line segments increases by a factor of four while the length of each segment is reduced by a third. So $n = 4$ and $s = ⅓$. Plugging these values into the formula gives the dimension for the Koch curve:

$$D = \frac{\log 4}{\log 3} = 1.2619$$

This is a fairly easy calculation, for the Koch snowflake is strictly self-similar because of its linearity. We can see this most clearly by plotting successive values s and $l(s)$ like Richardson did for coastlines and frontiers.[54]

Here,

$$s_n = \left(\frac{1}{3}\right)^n$$

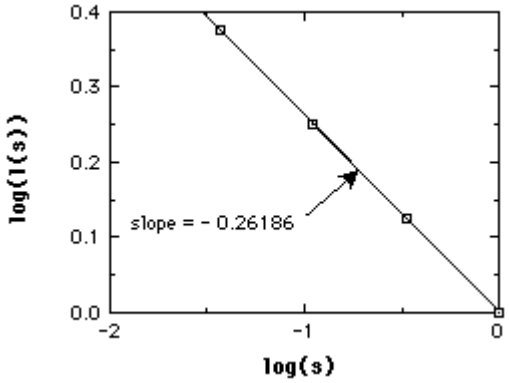
and

$$\log_{10} l(s) = b + a \log_{10} s$$

where

$$a = 1 - D = 1 - \frac{\log 4}{\log 3}$$

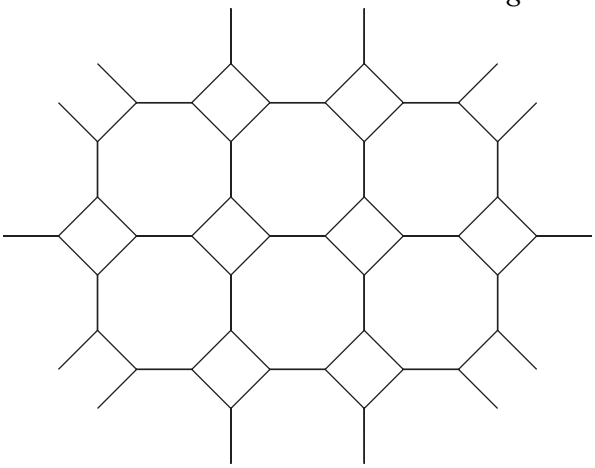
and $b = 0$ and $c = 1$ in Richardson's formulae.



However, repeating patterns in nature, mapped by nonlinear fractals, are not strictly self-similar. They are statistically self-similar, determined empirically rather than analytically, a tricky situation that does not need to concern us further. All we need to note is that fractal dimension provides a measure of the comparative 'densities' of fractals. In the case of the amazing Peano curve, this is a line that fills the plane and thus has dimension 2.

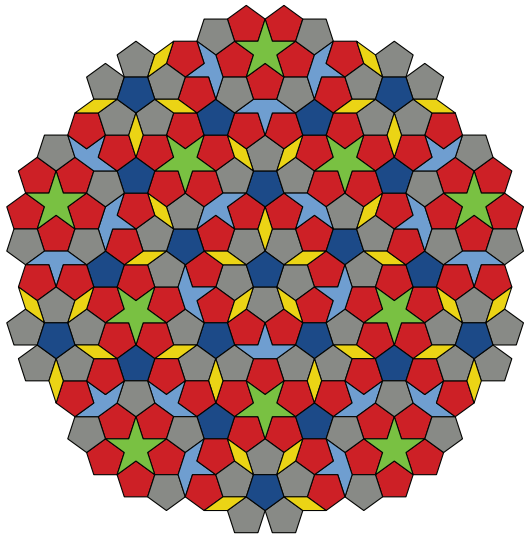
Today, fractal dimension is generally called Hausdorff dimension, occasionally called Hausdorff-Besicovitch dimension because Abram Samoilovitch Besicovitch (1891–1970) wrote three papers during the 1930s,[55] putting the Hausdorff dimension in 'final form', as Mandelbrot puts it.[56] For those interested in exploring further, Wikipedia has a large table for the Hausdorff-Besicovitch dimensions of many deterministic (linear) and random or natural (nonlinear) fractals. The dimension of the British coastline is given as 1.25.[57]

While we are on the subject of orthogonal structures being scaled, we can note that squares, for instance, are a pattern of tessellations that fill a flat surface, like Roman mosaics. For *tessellation* derives from Latin *tessellātus* 'of small square stones', from *tessella* 'small cube', diminutive of *tessera* 'a square', from Greek *tessares* 'four.' Triangles and hexagrams, which we see in bees' honeycombs, also do so. But why do bees use hexagrams when triangles or squares would do equally well? Well, Marcus du Sautoy, Simonyi Professor for the Public Understanding of Science at Oxford University, told us why in one of his many documentaries on the beauty of mathematics. A hexagrammatic structure encompasses the maximum amount of space for the minimum amount of material.

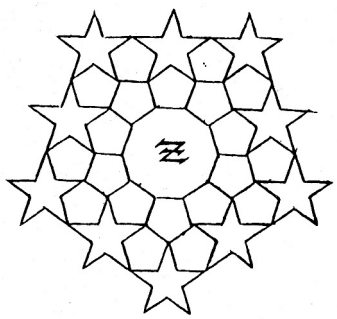


These three tessellations are formed from just one type of regular polygon. But nine others can be formed from two or three different polygons, with each vertex being the same. These are called semiregular or Archimedean, such as this one, which I have been using since the 1980s to illustrate the immense power of abstract thought once the repeating patterns underlying the Cosmos can be seen. This diagram depicts a seed that can grow to fill the Euclidean plane.

Such patterns do not need to be periodic. In the 1970s, Roger Penrose, a successor to Lewis Fry Richardson as the Rouse Ball Professor of Mathematics at Oxford University, developed what are today called Penrose tilings, such as this one.[58] This tiling is self-similar and nonperiodic (it lacks translational symmetry), yet covers the entire infinite plane. Penrose was inspired to create such tilings from Kepler's endeavours to create tessellations with pentagons, pentagrams, and decagons in *The Harmony of the World* in 1619.

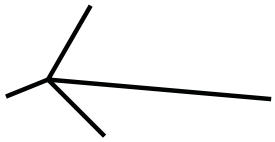


This problem arises because only regular trigons, tetragons, and hexagons have internal angles that are integral divisions of 2π radians or 360°. In contrast, the internal angle of a pentagon, which displays the golden ratio along with the pentagram, is 108° or 3π/5 radians. Kepler did manage to create this finite tiling, but had to cheat a little in creating an infinite tiling with such figures.[59]

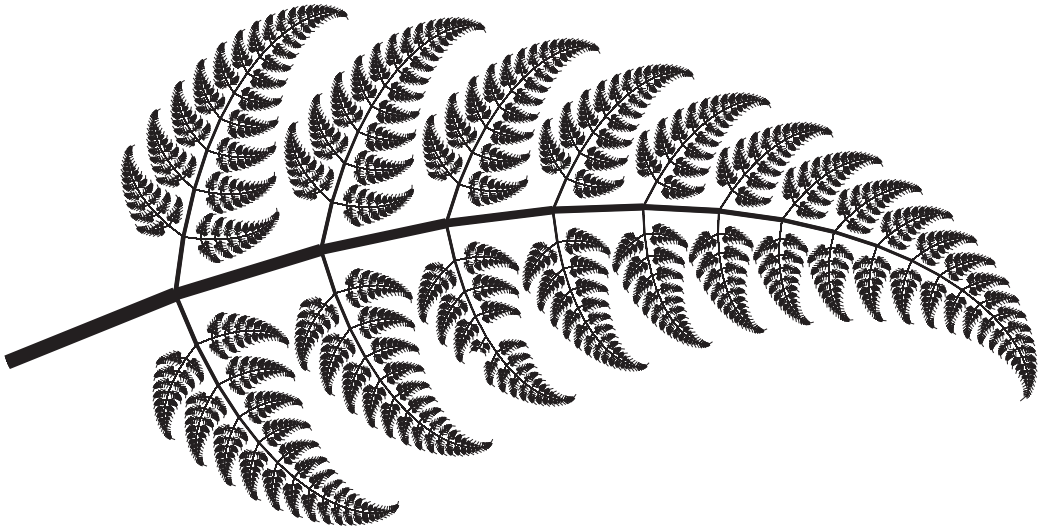


Kepler's *Harmonice Mundi*, in which he sought to find the beautiful unifying patterns underlying geometry, music, poetry, architecture, and astronomy, was a successor to his *The Six-Cornered Snowflake* of 1611, in which he asked why snowflakes have six corners, yet we see fives in many growth processes. As he observed, flowers on trees and bushes unfold in five-sided patterns, with five petals. These are in three dimensions, where pentagonal symmetry is quite possible. For, as Kepler pointed out, the dodecahedron and icosahedron "cannot be formed without the divine proportion [golden section] as modern geometers call it.[60] He then went on to discover the small and great dodecahedrons, first published in *The Harmony of the World*.[61]

We humans have been creating such mathematical and artistic patterns with different types of symmetry and asymmetry since antiquity. The possibilities are endless, where mathematics, nature, and spirituality meet, popular today as sacred geometry. Some of the most striking are the Islamic patterns to be found in Cordoba, Spain, as can be seen from a search of the Web.



But let us more on. To illustrate the way that a seed can grow into the beautiful structures we recognize in nature, in the mid 1990s, when I began to study fractals, I illustrated this point with that of a fern, modifying a Postscript program I found in *Exploring Fractals on the Macintosh*. Here is the seed, which grows into this picture of a fern after twenty-eight iterations (I don't yet know why there is a flaw in this version.)

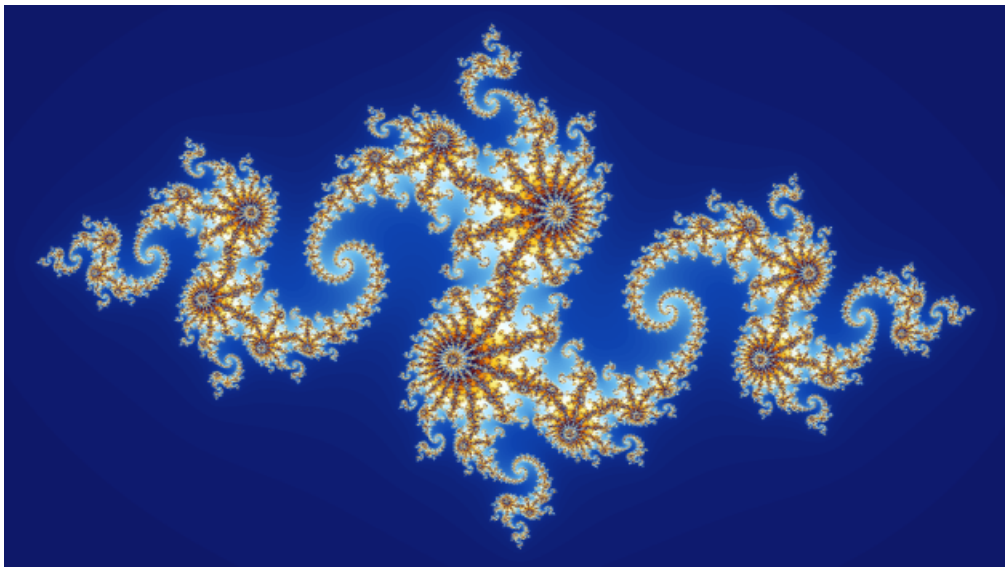


Returning to the origin of fractals as nonlinear, dynamic structures, the French mathematician Gaston Julia (1893–1978) explored this recurrence relation, where z_n and c, a constant, are complex numbers, such as $x + iy$ and $a + ib$:

$$z_{n+1} = z_n^2 + c$$

This formula is the same from which Mandelbrot created his eponymous set in the 1970s. It is deceptively simple, but with some very surprising results. In the case of Julia sets, for any given c, this is the set of *initial* values of z_0 for which the iterated values of z_n, called their orbit,[62] do not escape to infinity. What happens then is dependent on the absolute value $|z_0|$, which is $\sqrt{x^2 + y^2}$. In the special case of $c = 0 + 0i$, if $|z_0| < 1$, then these values are contained in a circle of radius one and z_n converges to the centre of the circle, called the point of attraction. If $|z_0| = 1$, then z_n remains on the boundary of the circle. And if $|z_0| > 1$, the orbit escapes to infinity.

But when c is complex, some really beautiful patterns emerge. For instance, if $c = -0.8 + 0.156i$, then the Julia set for z_0 looks like the picture below. Of course, when Julia conducted his studies, he did not have the benefit of computer graphics in which to present the multitude of different patterns that emerge. He would only have had a partial visualization of the patterns, which also must have been laborious to calculate.

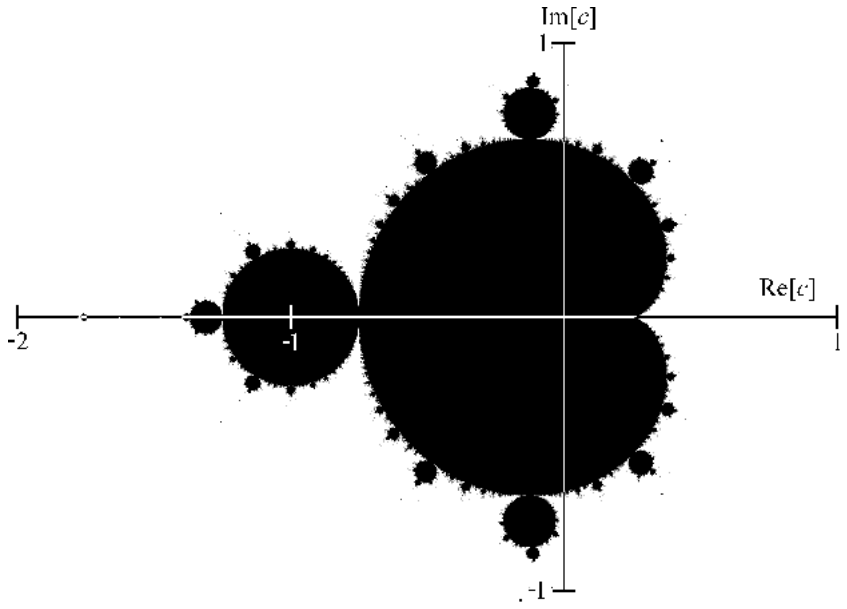


In the case of the Mandelbrot set, z_0 is zero and the difference equation is a function of c only. So there is only one Mandelbrot set, while there are an infinite number of Julia sets. Every point in the Mandelbrot set corresponds to a Julia set. In other words, the Mandelbrot set is a map for all the corresponding Julia sets. But more than this. A Julia set exists for all points in the complex plane, its characteristics depending on whether c is contained in the Mandelbrot set, near the boundary, or outside the set. In the first case, the Julia set is connected and in the third, it forms disconnected particles. Near the boundary, it is difficult to determine which.[63]

Like the Julia set, the Mandelbrot set is defined as those values of c that do not escape to infinity, which converge on an attractor. This is a characteristic of many nonlinear difference equations, including those that apply to evolution, as it passes through its Accumulation Point. The three basic attractors for the orbits of the Mandelbrot set are:

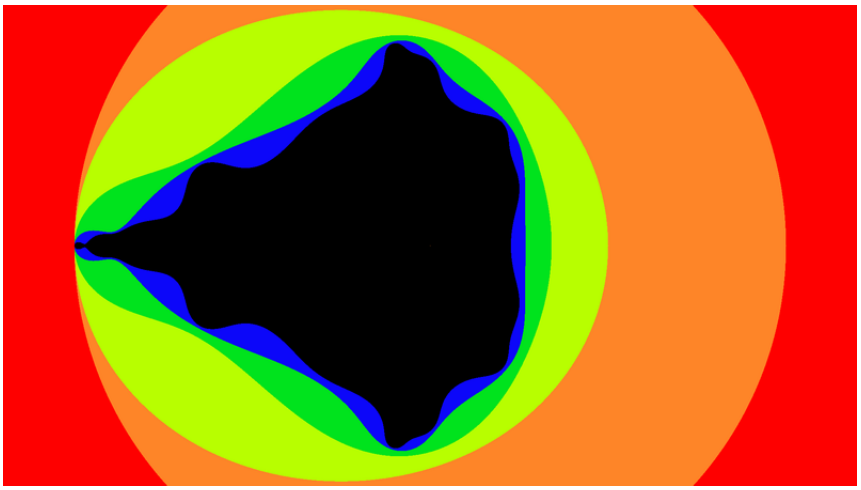
- converges to a single point
- cycles between two or more points
- randomly bounces about within a contained region[64]

These attractor points are usually shown as black in pictures of the set, not distinguishing between the different types of containment, which happens in chaos diagrams. Those outside the set tend to infinity at varying rates of acceleration, which are often depicted in different colours on the edges of the boundary of the set. Whether a set is in or out depends on the number of iterations. This diagram shows the outer bounds of the set, showing where the familiar cardioid and circular shapes relate to the real and imaginary axes. They are connected at -¾ on the real or x-axis.



Starting with these bounds, the next diagram shows what happens to z after 1 to 5 iterations.[65] The red area marks values of $|z_0| > 2$, giving $|z_1| > 2$. The orange circle has radius 2, marking values of $|z_0| < 2$ when $|z_2| > 2$. In turn, the yellow, green, and blue areas mark when $|z_3|$, $|z_4|$, and $|z_5| > 2$, respectively. Interestingly, the set is pinned at -2, for when c = -2, $z_{n+1} = 2^2 - 2$, for all $n \geq 2$.The position of the cusp of the cardioid is not so precise. A few trials in Excel reveals that it lies between 0.251 and 0.252. It should be quite easy to be more accurate with a Python program.

For values of c near the boundary of the set, it takes many more iterations to escape to infinity, for $|z_0|$ to become greater than 2. The set becomes more and more refined after 5, 10, 20, 50, 100 and 2000 iterations, although even after fifty iterations, the overall shape of the set is reasonably clear.[66] However, it



is not points in the set that humans find attractive. Rather, it is those near the border, which take a varying number of iterations to leave the set, which determine the colour of the pixel on the computer display or drop of ink on the printed page.

Initially, the Mandelbrot set is drawn at small-scale, like a map showing a large area at a scale of 1:1 000 000, with comparatively little detail, such as the Ordnance Survey wall map of Great Britain.[67] The number of pixels on the display determines the level of detail. This is also like using a ruler marked with only tenths of a centimetre, one decimal place. Then, when the scale is increased in smaller regions that are near the border of the set, we see the set in more detail, with more decimal places of precision.

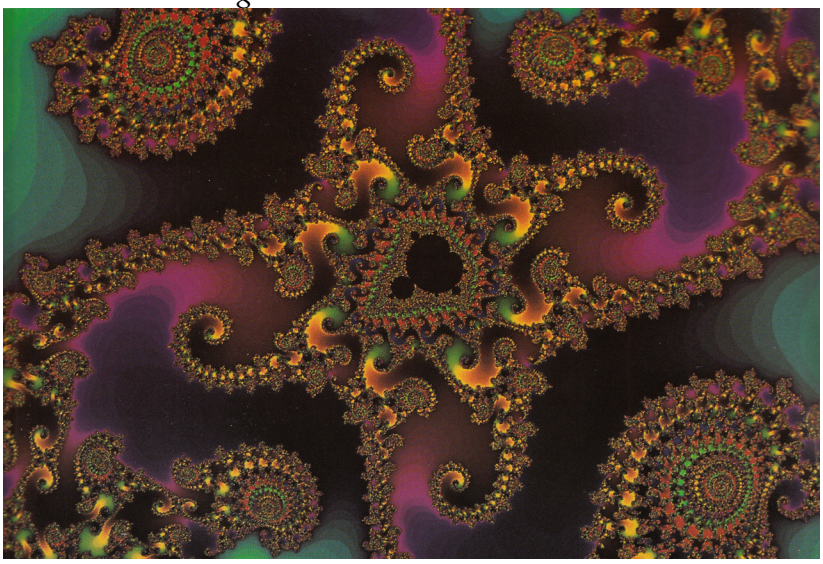
This is like the succession of maps that the Ordnance Survey produces, such as the OS Travel Maps of Scotland and North & Mid Wales at scales 1:500 000 and 1:175 000, respectively. To obtain more detail, the Landranger and Explorer leisure maps are mostly published at the scales of 1:50 000 and 1:25 000, respectively, such as those for Aberdeen and North York Moors. The OS MasterMap for business and government purposes is at the maximum resolution, containing 450 million geographic features at a scale of 1:1 250.[68] Google Maps and other digital maps used in our computers, tablets, and GPS devices, contain similar scaling options.

In the case of the Mandelbrot set, there are no physical limits on what scales can be displayed other than that of time. Multitudes of beautiful examples of quasi self-similarity are then revealed, as we see in such books as *The Beauty of Fractals*, by Heinz-Otto Peitgen and Peter H. Richter, and *Fractals: The Patterns of Chaos*, by John Briggs. Even the basic cardioid-circle shape re-appears in regions of great precision, as in the following jewel-box example that Rollo Silver created, which also almost displays rotational symmetry, quite fascinating. It is not surprising that the Mandelbrot set has become the symbol for the Chaos Revolution, as Briggs observes.[69]

What we have seen in this brief overview of fractal geometry are a number of patterns that are universal, essentially because we live in a holographic, self-similar Cosmos. Interestingly, poets, as harmonizing artists, encapsulated this universal sense of wholeness before scientists, as analysts, dividing the indivisible into fragments. For instance, in 1733, in *Poetry, A Rhapsody*, Jonathan Swift (1667–1745) observed scientists observing their external natural world. Then about 1803, in *Auguries of Innocence*, published in 1863, William Blake (1757–1827) wrote this oft-quoted opening stanza.

So, naturalists observe, a flea
Has smaller fleas that on him prey;
And these have smaller still to bite 'em.
And so proceed ad infinitum.

To see a world in a grain of sand,
And a heaven in a wild flower,
Hold infinity in the palm of your hand,
And eternity in an hour.



In 1872, in *Budget of Paradoxes*, Augustus De Morgan (1806–1871) paraphrased Swift's poem with these 'immortal' words:

> Great fleas have little fleas upon their backs to bite 'em,
> And little fleas have lesser fleas, and so *ad infinitum*.
> And the great fleas themselves, in turn, have greater fleas to go on;
> While these again have greater still, and greater still, and so on.

As I described in *The Theory of Everything* in 2014, between 1850 and 1860, De Morgan introduced the logic of relations into mathematical logic, which Charles Sanders Peirce (1839–1914) further developed in the 1880s.[70] This led Edgar F. 'Ted' Codd (1923–2003) of IBM to develop the relational model of data in the late 1960s, lying at the heart of all databases on the Internet.[71] As this modelling method introduced a nondeductive approach to mathematical logic, it allows Integral Relational Logic to admit self-contradictions into reasoning in an entirely valid manner, showing that the entire Totality of Existence is semantically self-similar, expressed in this statement: *The underlying structure of the Universe is an infinitely dimensional network of hierarchical relationships*.

Returning to quantitative mathematics, when studying convection currents in the atmosphere, Lewis Fry Richardson quoted this observation that C. K. M. Douglas made from an early aeroplane, presumably open to the skies, "The upward currents of the cumuli give rise to turbulence within, below, and around the clouds, and the structure of the clouds is often very complex." Richardson was thus inspired to paraphrase De Morgan's little ditty with these words: "big whirls have little whirls that feed on their velocity, and the little whirls have lesser whirls and so on to viscosity—in the molecular sense."

Regarding fractals, in 1990, Jasper D. Memory wrote this poem on 'Blake and Fractals' published by the Mathematical Association of America,[72]

> William Blake said he could see
> Vistas of infinity
> In the smallest speck of sand
> Held in the hollow of his hand.
>
> Models for this claim we've got
> In the work of Mandelbrot:
> Fractal diagrams partake
> Of the essence sensed by Blake.
>
> Basic forms will still prevail
> Independent of the Scale;

Viewed from far or viewed from near
Special signatures are clear.

When you magnify a spot,
What you had before, you've got.
Smaller, smaller, smaller, yet,
Still the same details are set;

Finer than the finest hair
Blake's infinity is there,
Rich in structure all the way–
Just as the mystic poets say.

In 1984 and 1990, John P. Briggs and F. David Peat gave excellent summaries of the convergence of chaos theory with relativity and quantum theories as they were understood at the time in *Looking Glass Universe: The Emerging Science of Wholeness* and *Turbulent Mirror: An Illustrated Guide to Chaos Theory and the Science of Wholeness*. Bringing these books right up to date, this essay shows that there is a direct connection between the Mandelbrot set and the future of humanity. But before we explore what this means for population growth as evolution accelerates through its Accumulation Point, we need to explore a little what Swift and De Morgan meant by *ad infinitum*.

Large exponential numbers

But before we look at the way numbers head to infinity and the Transfinite Divine at evolution's Glorious Culmination, I find it useful to obtain a deeper intuitive understanding of how large exponential numbers can rapidly become. As the physicist Albert A. Bartlett has said, "The greatest shortcoming of the human race is our inability to understand the exponential function."[73] We have some sense of what 100 years is like or even 1000, measured from our own lifespan of threescore years and ten, as the Psalmist put it.[74] But what is a billion years in our experience or a quintillion years? Our lack of understanding of large numbers is well illustrated by a story that is said to have originated from the invention of chess.

> According to an old tale, the Grand Vizier Sissa Ben Dahir was granted a boon for having invented chess for the Indian King, Shirham. Since this game is played on a board with 64 squares, Sissa addressed the king: 'Majesty, give me a grain of wheat to place on the first square, and two grains of wheat to place on the second square, and four grains of wheat to place on the third square, and eight grains of wheat to place on the fourth square, and so, Oh King, let me cover each of the 64 squares on the board.' 'And is that all you wish, Sissa, you fool?' exclaimed the astonished King. 'Oh, Sire,' Sissa replied, 'I have asked for more wheat than you have in your entire kingdom, nay, for more wheat than there is in the whole world, verily, for enough to cover the whole surface of the earth to a depth of the twentieth part of a cubit.'[75]

The reason for this is that the number of grains of wheat on the nth square is 2^{n-1}. And $1 + 2 + 4 + \ldots + 2^{63}$ is $2^{64} - 1$, equal to 18,446,744,073,709,551,615, about 18 quintillion. This is one less than the theoretical storage capacity of modern 64-bit processors, such as the iMac I am using to write this essay. But this has a storage capacity of just 16 gigabytes, 2^{34} and the most powerful Mac that Apple sells, the Mac Pro, has a maximum memory of 64 gigabytes, 2^{36}. These numbers are far smaller than the theoretical maximum for such processors. The number 2^{64} also appears in another story from antiquity:

> In the great temple at Benares, beneath the dome which marks the centre of the world, rests a brass plate in which are fixed three diamond needles, each a cubit high and as thick as the body of a bee. On one of these needles, at the creation, God placed sixty-four discs of pure gold, the largest disc resting on the brass plate and the others getting smaller and smaller up to the top one. This is the tower of Brahma. Day and night unceasingly, the priests transfer the discs from one diamond needle to another, according to fixed and immutable laws of Brahma, which require that the priest on duty must not move more than one disc at a time and that he must place this disc on a needle so that there is no smaller disc below it. When the sixty-four discs shall have been thus transferred from the needle on which, at the

creation, God placed them, to one of the other needles, tower, temple, and Brahmans alike will crumble into dust, and with a thunderclap, the world will vanish.[76]

Now it turns out that the number of transfers that the priests would need to make is again $2^{64} - 1$. If the priests were to make one transfer every second, and work 24 hours a day for every day of the year, it would take them 584,542,046,091 years to perform this feat, 11 orders of magnitude or about 40 times longer than the time since the most recent big bang. And if we measure time in yoctoseconds or septillionths of a second, (10^{-24}), the shortest unit of temporal measure that I am aware of, the most recent big bang happened about 4×10^{41} yoctoseconds ago, just yesterday in the cosmic scale of things.

As another illustration of exponential power from the world of technology, when IP addresses for Internet Protocol were first set up, they consisted of four groups of 2^8 bits (256) giving 2^{32} possible values. Since then, the Internet has expanded so fast that this IPv4 address space has become exhausted. The Internet Engineering Task Force (IETF) has accordingly defined an IPv6 address space of 2^{128} possible values (about 3.403×10^{38} or 340 undecillion), which should last quite a while, even if only the square root (2^{64}) or half the power seems to be defined as four groups of four hexadecimal digits.[77]

Similarly, in spiritual circles, monks, who wrote the *Avatamsaka Sutra* '*Flower Ornament Sutra*' in Huayan Buddhism, began Chapter Thirty when they contemplated the Incalculable with these words:

> At that time, the enlightening being Mind King said to the Buddha, "World Honoured One, the buddhas speak of incalculable, measureless, boundless, incomparable, innumerable, unaccountable, unthinkable, immeasurable, unspeakable, untold numbers—what are these?"
>
> The Buddha said, "It is good that you ask the Buddha, the Truly Enlightened One, in order to have the beings of the world penetrate the meaning of the numbers known to the Buddha. Listen carefully and think well about this; I will explain for you."

The Buddha then delineated an exponential series, whose first three terms are, "Ten to the tenth power times ten to the tenth power equals ten to the twentieth power; ten to the twentieth power times ten to the twentieth power equals ten to the fortieth power; ten to the fortieth power times ten to the fortieth power equals ten to the eightieth power." In other words, the first term in the series is 10^{20} or 100 quintillion, the fifth root of a googol, and each succeeding number is the square of the previous one. In mathematical terms:

$$a_n = a_{n-1}^2 = 10^{10 \times 2^n}$$

However, the Buddha stopped when n = 103, which is 10 to the power of 100 nonillion approximately. The book says that this is ten to the power of 101,493,292,610,318,652,755,325,638,410,240, although there is an error in the calculation. It should be 101,412,048,018,258,352,119,736,256,430,080. According to the Sutra, the Buddha then said that a_{103}^2 is 'incalculable', calling the next few terms 'measureless', 'boundless', 'incomparable', 'innumerable', 'unaccountable', 'unthinkable', 'immeasurable', 'unspeakable', and 'unspeakably unspeakable', proceeding in fourth powers rather than second for he was now in a hurry. He ended by saying, "an untold, which is unspeakably unspeakable, … multiplied by itself, is a square untold".[78] This is a_{123}, nevertheless comprehensible to the human mind.

In temporal terms, periods in the ancient Hindu calendar represent the cyclic changes that society as a whole goes through over time, within the time-cycle of the creation and destruction of the Universe, denoted by the life span of Brahma, the god of creation. The basic unit of these cycles is the *mahayuga*, consisting of four *yugas* of diminishing time periods, measured in divine years, each of which is 360 human years, although some writers refer to the four cycles collectively as a yuga.[88] This table shows these time periods, which diminish in arithmetic progression.

Name	Characteristic	Years	Divine years
Krita- or Satya-Yuga	Golden age	1,728,000	4,800
Treta-Yuga	Sacrifices begin	1,296,000	3,600
Dvarpara-Yuga	Spiritual decline	864,000	2,400
Kali-Yuga	War, fear, and despair	432,000	1,200
Mahayuga		**4,320,000**	**12,000**

In Hindu attempts to capture the vastness of time before the discoveries of modern mathematics, Brahma is deemed to live 100 Brahma-years, to denote the creation and death of the universe, each Brahma-year consisting of 360 Brahma-days and nights or 720 *kalpas*. So in the Hindu calendar, the life and death cycle of the Universe is 311 trillion years, or 14 orders of magnitude.

This is quite small compared to the lifespan of the physical universe presented by Brian Cox in the 'Destiny' episode of his BBC documentary series *The Wonders of the Universe* in 2011. Apparently drawing on Fred Adams and Gregory Laughlin's *The Five Ages of the Universe*,[79] he said that the physical universe has a lifespan of "10,000 trillion trillion trillion trillion trillion trillion trillion trillion trillion years", which is 10^{100}, just one googol, defined below.

We can see from these examples that we can much better understand the power of exponential numbers if we think in terms of orders of magnitude or powers of ten, rather than the numbers themselves.

However, we can see from the *Flower Garden Sutra* that Buddhists do not actually have a limited view of time, as do Hindus, astrophysicists, and Christian fundamentalists. In Buddhism, *kalpa* is a "term for an endlessly long period of time, which is the basis of Buddhist time reckoning. The length of a *kalpa* is illustrated by the following metaphor: suppose every hundred years a piece of silk is rubbed once on a solid rock one cubic mile in size; when the rock is worn away by this, one *kalpa* will still not have passed away."[80]

Although Buddhists did not have a mathematical way of representing such vast periods of time, mathematicians have created notations for unbelievably large exponential numbers. For instance, in the 1930s, Edward Kasner tried to explain exponential numbers to his nine year-old nephew Milton Sirotta by asking him to create a name for a very big number.[81] Milton showed that he had more wisdom than his mathematician uncle imagined. For he coined the word *googol* for 10^{100},[82] which is just 100 orders of magnitude, still quite manageable by the mind. In 1997, Google adapted this term, a misspelling of *googol*, for its search engine, to denote its mission to organize a seemingly infinite amount of information on the web.[83] However, Milton went even further, also defining a googolplex as 10^{googol}, a name that Google has given to its headquarters. But what on earth is a googol orders of magnitude? Or any of Gödel's proof numbers in his incompleteness theorem?[84] Or a googolplex to the power of a googolplex three times, like this, a number that is quite beyond our imagination:

$$googolplex^{googolplex^{googolplex^{googolplex}}}$$

Infinity of infinities

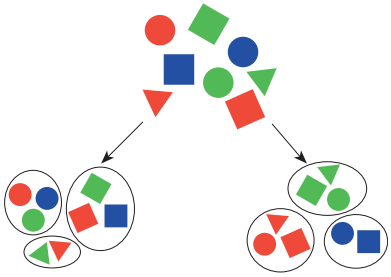
Yet, even raising a googolplex to the power of a googolplex googolplex times is tiny compared with the smallest infinity, the count of the integers or rationals, or the 'largest' infinity. The mathematical concept of infinity went through a major development in the last quarter of the 1800s, when Georg Cantor (1845–1918) developed the mathematical theory of sets, the most fundamental concept in mathematics, and, indeed, in all of human learning.

To Cantor, the concept of set was essentially intuitive, defined as follows: "By a set we mean the joining into a single whole of objects which are clearly distinguishable by our intuition or thought,"[85] a definition that I discovered in 1980, when setting out to explain what is causing scientists and technologists to drive the pace of scientific discovery and technological invention at exponential rates of acceleration. However, in writing this essay on the mathematics underlying the holistic theory of evolution that has since evolved, I have found Cantor's more formal definition given in a synoptic work of 1895: "By a set we are to understand any collection into a whole *M* of definite and distinguishable objects of our intuition or our thought. These objects are called the elements of *M*."[86]

Sets are thus fundamental to bringing our thoughts into universal order and, as such, need some clarification, a quality that it is of the utmost importance to mathematicians, just as it is to information systems architects, philosophers, and lawyers. We can see why mathematics is regarded as the archetype of conceptual clarity from this mathematical joke:

> An astronomer, a physicist, and a mathematician (it is said) were holidaying in Scotland. Glancing from the train window, they observed a black sheep in the middle of the field. 'How interesting,' observed the astronomer, 'all Scottish sheep are black!' To which the physicist responded, 'No, no! Some Scottish sheep are black!'. The mathematician gazed heavenward in supplication, and then intoned, 'In Scotland there exists at least one field, containing at least one sheep, at least one side of which is black.'[87]

A difficulty with Cantor's definition of set, which I have only just noticed, as I write these words in February 2016, is that it appears that the elements of a set are distinguishable from each other; elements are unique. Yet, in forming such a set, we could also think of the elements in sets as distinguishable from those in other sets, being differentiated by some property that the elements have in common. This appears to be what Cantor meant by the wholeness of a set.



We can see this most clearly by this illustration of bringing our thoughts into order. Objects of various shapes and colours can be grouped into sets labelled 'triangle', 'square', and 'circle' or 'red', 'green', and 'blue'. This is an example of what David Bohm (1917–1992) meant by bringing order to quantum physics. As he pointed out, we can bring universal order to our thought processes by "*giving attention to similar differences and different similarities*", a notion of order that the artist Charles Biederman had given him.[88]

Having formed such sets, we can then form the concept of number. For, in this example, two sets contain three members, while one has just two, giving a property by which sets, themselves, are members in other sets. The numeral III, for instance, represents a set containing sets of three elements. Even though human beings had been using numbers for thousands of years before, they were not aware that you cannot actually form the concept of number, and hence number 3, without the concept of set. Sets are more fundamental concepts than those of numbers, both in mathematics and in concept formation, in general.

In other words, semantics, the science of meaning, is more fundamental than mathematics, as the science of space and number, which became the more general science of patterns and relationships after George Boole freed mathematics from the tyranny of number systems in 1844 with his operator theory. This was published as 'On a General Method in Analysis' in the *Philosophical Transactions of the Royal Society of London*,[89] for which he was awarded the Royal Society's first gold medal for mathematics, known as the Royal Medal,[90] even though he had left school at sixteen and did not have a degree.

Drawing on Duncan F. Gregory's generalizing principles, Boole regarded the essence of mathematics as "the study of form and structure rather than content, and that 'pure mathematics' is concerned with the

laws of combination of 'operators' in their widest sense." For instance, he noted that the commutative and distributive laws of arithmetic could equally apply to differential operators and geometric transformations.[91]

Recognizing that the concept of set provides the cognitive foundation for mathematics and logic, in the 1960s, some mathematicians introduced the abstract concept of set into primary and elementary schools, so that children could intelligently and consciously learn how to form concepts, like the illustration above.

This transcultural, transdisciplinary interpretative process is central to pattern recognition, conscious evolution, and all our learning. As the authors of *The 'New' Maths* pointed out, the new maths was intended to bring meaning to mathematics and hence to all other disciplines.[92] However, this initiative was then abandoned because it did not meet the numerical needs of science and business. So many children leave school today without knowing that sets lie at the foundation of rational thought, although they intuitively understand the concept when it is explained to them as adults.

This might seem reasonably straightforward. But if you pick up a book on the mathematical theory of sets, you will find a very different story. For instance, the first chapter of Thomas Jech's 765-page scholarly tome *Set Theory* begins with nine axioms of mathematical set theory.[93] Not recognizing that sets are first and foremost semantic concepts, in 1908, Ernst Zermelo (1871–1953) defined an initial collection of axioms for set theory. To avoid the paradoxes inherent in set theory, his purpose was to clarify what is meant by a set and what properties it should have. However, in 1922, Abraham Fraenkel (1891–1965) pointed out that Zermelo had failed to distinguish the property of a set and the set itself, suggesting an amendment.[94] So the axioms of set theory are today known as the Zermelo-Fraenkel axioms, the first eight being denoted by ZF.

The ninth axiom is called the 'axiom of choice', which is a rather strange construct. It is the assertion that, given any collection of sets, finite or infinite, one can select one object from each set and form a new set. Here is an example that Morris Kline gives in *Mathematics: The Loss of Certainty*, "from the people in all the fifty states of the United States one can pick one person from each state and form a new set of people".[95] Zermelo had pointed out in 1904 that many mathematical proofs implicitly depend on the axiom of choice, so it needs to be explicitly included in the axioms of set theory.

However, the existence of such sets in the foundations of mathematics created an uproar, for, as Bertrand Russell pointed out, a set is defined by a property that all members possess. So the axiom of choice is not a choice at all. There is no attribute that defines the members of such a set other than that they have been arbitrarily selected from different sets. Nevertheless, for those proofs that are dependent on the axiom of proof, this ninth axiom is included in the Zermelo-Fraenkel axioms, which are known as ZFC.

In contrast to Jech's indigestible tome, Azriel Levy began *Basic Set Theory* with this sentence: "All branches of mathematics are developed, consciously or unconsciously, in set theory or in some part of it." As he pointed out, this is not a problem for those who apply set theory in their studies. However, "In developing set theory itself we have no such advantage and we must go through the labour of setting up our set theoretical apparatus." Most significantly, set theoreticians require a language and notation in which to describe the mathematical theory of sets. Levy chose to solve this problem by using first-order predicate calculus with equality.[96]

From these basic assumptions, Jech and Levy then present mathematical set theory in their textbooks, describing a host of different types of sets and their properties in an extremely complex manner without

being aware of the simplicity underlying all our learning. In this respect, they demonstrate the growth of mathematical structures, following the traditional linear path that Euclid described in *The Elements*, the systematization of much of the mathematics known to the Ancient Greeks. For Euclid, mathematics begins with a few self-evident or assumed axioms or postulates that do not need proving because they are self-evident. Thereby, he laid down the foundations of mathematical axiomatic method, which holds sway, even today.

We can discover the simplicity underlying the axioms of mathematics and, indeed, all knowledge, by starting at the Absolute Truth, rather than some assumed truths in the relativistic world of form, which can lead us perilously astray. As mentioned in the Introduction and described in some detail in my other writings, I call the Absolute the Datum of the Universe, the Ultimate Giver and Cause of everything that exists.

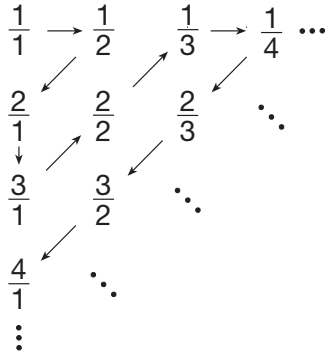
By starting afresh at the very beginning in this way, we can become liberated from our mechanistic cultural conditioning, living predominantly in the vertical dimension of time—in the Eternal Now—rather than having our thoughts and actions determined by the past, projected into the future. We then live in complete harmony with the Divine, following its energies moment by moment, with as little interference from the egoic and anthropocentric mind as possible.

◇

For me, understanding how the infinity of infinite sets leads to God, as the Transfinite Absolute, helps with this liberating and awakening process. So what is a set for those without a mathematical background? Well, as we have seen, it is just a group or collection of items with a common property, such as shape or colour, at the heart of the concept of self-similarity in fractals. With this straightforward concept of set, Cantor then set out to count the number of members in infinite sets, developing the concept of countable set, and got some amazing results.

A set is countable if you can count its objects. Fairly obviously, a finite set is countable. An infinite set is also regarded as countable if all the elements in the set can be put into sequential order in one-to-one correspondence with the natural numbers: positive integers, which may also include zero. For instance, the even and odd integers and even all the rationals are countable. To see this, we simply use a mapping like this, where n is any integer: $2n \rightarrow n$.

This was not a new idea. In *Dialogue Concerning the Two Chief Systems of the World*, published in 1632, Galileo Galilei (1564–1642) made this telling observation, "There are as many squares as there are numbers because they are just as numerous as their roots."[97] In other words, $n^2 \rightarrow n$, like Stifel's mapping in 1544 on page 7, which led to the discovery of logarithms. However, Galileo got into deep trouble with the Catholic authorities with this book for claiming that he had 'conclusive physical proof' for the heliocentric worldview, not having read Kepler's *New Astronomy*, which *did* provide the conclusive proof that he could have used if he had been a little less arrogant.[98]



It is not difficult to prove that the rational numbers, of the form p/q, map to the integers, even just those that lie between 0 and 1. This diagram illustrates the way that all the rationals can be arranged in a sequence that maps to the integers: 1/1→1, 1/2→2, 2/1→3, 3/1→4, 2/2→5, 1/3→6, and so on. Yes, rationals in which the numerators and denominators are not coprime are duplicated, but this doesn't matter; it merely serves to emphasize the point. This mapping shows that the count of rationals is the same as that of the integers, even

though there are an infinite number of rationals between any two rationals. Indeed, there are an infinite number of points between any two points in the Koch snowflake, formed stepwise, like the series of integers. Surprisingly, the sets of positive even integers, all integers, and rationals have the same level of infinity. They are all countable, in Cantor's terms, leading to some very strange arithmetic, like this: $\infty + \infty = \infty$.

But what about the algebraic numbers? These are defined as the roots of a polynomial equation

$$a_0x^n + a_1x^{n-1} + \cdots + a_{n-1}x + a_n = 0$$

where $a_0 \neq 0$, n is a natural number, and each a_i is an integer. In a famous paper published in 1874, which launched Cantor's studies of infinite sets, he proved that algebraic numbers, like $\sqrt{2}$ and φ, the Golden Ratio, are countable. He did this with the concept of the 'height' of an algebraic equation, defined as

$$h = n + a_0 + |a_1| + \cdots + |a_{n-1}| + |a_n|$$

For instance, $3x^2 - 2x + 1 = 0$ has height $h = 2 + 3 + 2 + 1 = 8$. David M. Burton then explains Cantor's proof in this way:

> For any fixed height h, the integers n, a_0, a_1, …, a_{n-1}, a_n can be specified in only a finite number of ways, thereby leading to a finite number of equations; each such equation can have at most as many different roots as its degree. Thus, there are just a finite number of algebraic numbers arising from equations of a given height. By grouping the algebraic equations according to height, starting with those of height 2, then taking those of height 3, and so on, one can write down the set of algebraic numbers in a sequence.[99]

But are all real numbers algebraic? Do numbers exist that are not roots of an algebraic equation, called 'transcendental', for as Euler said, "They transcend the power of algebraic methods."[100] This question caused a lot of botheration in mathematical circles until Joseph Liouville (1809–1882) proved in 1844 that numbers of this form are transcendental:

$$\sum_{n=1}^{\infty} \frac{a_n}{10^{n!}}$$

where a_n is an arbitrary number between 1 and 9, inclusive. For instance, if all the a's are 1, this strange number, consisting mostly of zeroes, is transcendental: 0.11000100000000000000000010….[101] But what about e and π? Are they algebraic or transcendental? Well, it is not easy to prove whether any particular number is algebraic or not. In the event, it was not until 1873 that Charles Hermite proved that e is transcendental. Then nine years later, C. L. F. Lindemann (1852–1939) proved that π is transcendental.[102]

In the meantime, Cantor had proved that it does not really matter whether any particular number is transcendental, for their number far exceeds the count of the algebraic numbers. They are not countable, not mappable to the integers. He did this using what is known as the 'diagonal method'. The method is by indirect proof, which George Pólya (1887–1985) likened "to a politician's trick of establishing a candidate by demolishing the reputation of his opponent", related to the method of *reductio ad absurdum*, which Pólya likened to the irony of a satirist, stressing and overstressing a point until it leads to an absurdity.[103]

Let us assume that the real numbers, consisting of all algebraic and transcendental numbers, are countable. In that case, it is possible to map the reals strictly between 0 and 1 to the integers, expressing nonterminating decimals as infinite decimals, such as 0.333… and 0.4999… for ⅓ and ½, respectively. Such a set, arranged in denumerable or countable order, could look like this:

$$\begin{aligned}
1 &\leftarrow 0.a_{11}a_{12}a_{13}\ldots \\
2 &\leftarrow 0.a_{21}a_{22}a_{23}\ldots \\
3 &\leftarrow 0.a_{31}a_{32}a_{33}\ldots \\
&\vdots\ \vdots\ \vdots\ \vdots\ \vdots\ \vdots\ \vdots\ \vdots\ \vdots\ \vdots\ \vdots\ \vdots
\end{aligned}$$

where a_{ij} is a digit between 0 and 9 inclusive. In the words of Carl B. Boyer, who wrote a classic book on the history of mathematics, "Cantor then exhibited an infinite decimal different from all of those listed. To do this, simply form the decimal $b = 0.b_1b_2b_3\ldots$, where $b_k = 9$ if $a_{kk} = 1$ and $b_k = 1$ if $a_{kk} \neq 1$." This real number is different from all those in the list, which is supposed to include all reals. Hence the set of real numbers, which Cantor denoted with c for 'continuum', is not countable.[104] As this substitution is a binary one, the proof can be expressed in binary arithmetic, as is done in Wikipedia.[105]

Cantor did not stop there. In 1878, he published a paper that showed that the points in a finite square, 'clearly two-dimensional', can be put into one-to-one correspondence with the points of a straight line segment, 'clearly one-dimensional'.[106] Indeed, there is no need to stop at two dimensions. This mapping can be extended into the three dimensions of a cube and then into any number of dimensions. Furthermore, the points in a finite n-dimensional hypercube can be mapped into infinite 'space'. "Dimensionality is not the arbiter of the power of a set."[107] In other words, the cardinality of the points in all of n-dimensional space is c.

Cantor was so shocked by this result that in 1877 he wrote to his friend Richard Dedekind (1831–1916), "I see it, but I do not believe it." Not surprisingly, he had considerable difficulties in getting his revolutionary discoveries published, with Leopold Kronecker (1823–1891) being particularly hostile, leading the hypersensitive Cantor to suffer a series of 'nervous breakdowns' from 1884 to the end of his life, dying in a mental institution.[108]

However, with the authority of Dedekind to support him, Cantor's paper was published, allowing him to continue with his studies of infinite sets. He went on to prove some even more surprising results. There are not just two infinite cardinals; there are an infinite number of them, which Cantor called *transfinite*, which has a quite different meaning in Integral Relational Logic, as we see at the end of this subsection. He did this in two ways, using the concepts of power set and infinite ordinals.

First, Cantor proved that the power set of a set is larger than the set itself. For instance, the power set of {a b c} has eight members (2^3): {{a b c} {a b} {b c} {c a} {a} {b} {c} {}}. This property of sets also applies to infinite sets, such as countable ones, whose cardinality Cantor denoted by $\aleph_0$. "In particular, by considering all the possible subsets of the set of whole numbers, Cantor was able to show that $2^{\aleph_0} = c$."[109]

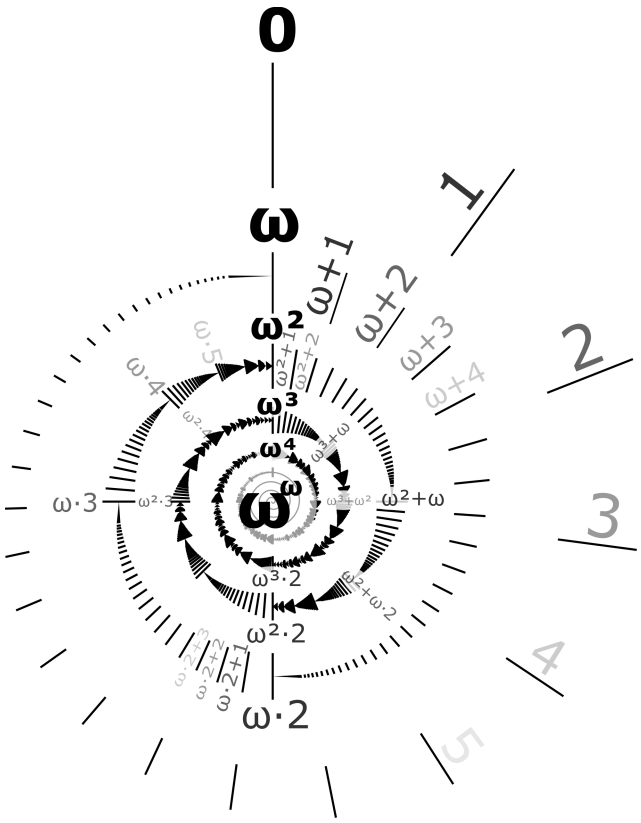
It is not only the set of real numbers that is uncountable. For instance, Roger Penrose has pointed out that there are $2^{\aleph_0}$ different patterns of nonperiodic tilings, such as the one on page 16. As he said, "Different arrangements are, in a certain 'finite' sense, all indistinguishable from each other. Thus, no matter how large a finite portion is selected in one such pattern, this finite portion will appear somewhere in every completed pattern (infinitely many times, in fact)."[110]

In turn, the cardinality of the power set of a set with c members is strictly greater that the cardinality of c, a sequence that can be continued indefinitely. It might seem natural to denote this sequence with $\aleph_0$, $\aleph_1$, $\aleph_2$, $\aleph_3$, etc. However, Cantor did not do this, leaving the infinite sequence of power sets unnamed, as far as I can tell.

Rather, he reserved these names for a sequence of members in infinite sets, called ordinals. These are formed in a rather strange way, whose meaning is not easy to grasp. However, infinite ordinals are most important, for they illustrate the amazing way in which numbers can grow in magnitude, actually representing the way that the mind works as an example of the patterns underlying the Cosmos. The key point in this context is that numbers can be used for two purposes: to denote the number of elements in a set and to denote the position of an element in an ordered sequence.

For finite sets, there is no difference in these two uses. The numbers 1, 2, and 3, etc. can be used, even if *first*, *second*, *third*, and so on denote this difference. However, when it comes to infinite sets, Cantor made a distinction. He denoted the ordinal of a well-ordered countable set with Greek omega, ω. But then he said that this ordinal has a successor ω + 1, which does not change the cardinality of the set. Infinite ordinals can be combined arithmetically with three operations, addition, multiplication, and exponentiation, which we do not need to look at in any detail.

In diagrammatic form, the first turn of the spiral in the diagram below depicts the first infinite series of ordinals, terminating at ω. Then, following the ω ordinal, there are a countable number of successor ordinals terminating at ω giving ω · 2 as the limit ordinal, depicted by the first half of the second spiral moving inwards. These are then followed by an infinitely countable series of an infinitely countable series of ordinals, which terminate at ω^2, completing the second spiral. As the exponent is an ordinal with a limit of ω, as the exponents increase towards ω, there are a countable series of countable series of countable series terminating at ω^ω.



Cantor did not stop there. Using a different notation, he continued to add ordinals to the countable infinite set, until he eventually reached the first uncountable ordinal, which is usually denoted by ω_1, with cardinality $\aleph_1$. Of course, this is not the end of the way that the mind can keep on growing numbers so that they become larger and larger. Using the set with cardinality $\aleph_1$ as a base, Cantor added countless ordinals to ω_1, leading to ω_2, with cardinality $\aleph_2$. Naturally, such a process can continue indefinitely. In mathematics, there is thus an infinite series of well-ordered sets with ordinals ω, ω_1, ω_2, ω_3, etc, with cardinality $\aleph_0$, $\aleph_1$, $\aleph_2$, $\aleph_3$, etc.[111]

The obvious question now to ask is whether the two infinite series of ordinals and power sets are equivalent. Specifically, does $\mathfrak{c} = \aleph_1 = 2^{\aleph_0}$? If so, "There is no set whose cardinality is strictly between that of the integers and that of the real numbers," as Cantor hypothesized in 1874.[112] Proving that the real number continuum is the smallest noncountable set is so important for mathematicians that it was the

first of 23 unresolved mathematical problems that David Hilbert posed at the International Congress of Mathematicians in Paris in 1900,[113] known as the continuum hypothesis.

Applying this hypothesis to the two ways of creating an infinite series of infinite cardinals, the generalized continuum hypothesis asks whether it is true that $2^{\aleph_n} = \aleph_{n+1}$? In the event, Kurt Gödel (1906–1978) in 1940[114] and Paul Cohen (1934–2007) in 1963[115] showed that the hypothesis can neither be disproved nor be proved using the axioms of Zermelo-Fraenkel set theory, provided these axioms are consistent.[116] Or we could say that the generalized continuum hypothesis is both unprovable and undisprovable through axiomatic, linear reasoning, one of countless instances of the universal Principle of Unity.

Nevertheless, for all practical purposes, it is reasonable to assume that the infinite series of power sets includes all distinct infinite cardinals, denoting the 'largest' infinity as $\aleph_\infty$, where ∞ is $\aleph_\infty$, defined recursively, ad infinitum! But what then is eternity or infinite time? And when people say that the soul is immortal, living for eternity, which infinity are they referring to?

Well, to answer this question, we need to look at other shocks that emerged from Cantor's set theory, which gave rise to several paradoxes. A simple example of a contradiction in set theory is the notion that we can form a set that includes all sets, which Cantor noted in 1895.[117] But he also pointed out that the power set of this set is larger this 'largest' set, a semantic contradiction. This and other paradoxes in the foundations of mathematics led Hilbert to define the second of his unsolved problems in mathematics in 1900: to prove that the axioms of mathematics are consistent, that they do not contain self-contradictions.

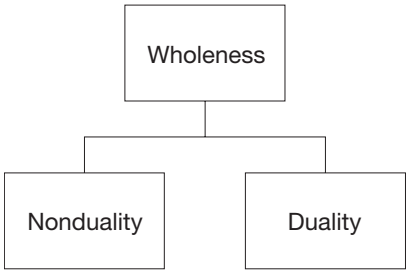
In particular, Bertrand Russell was so disturbed by the paradoxes and self-contradictions revealed in the foundations of mathematics that at the beginning of the last century he formulated the Theory of Types to eliminate them from formal logic,[118] futilely spending twenty exhausting years with Arthur North Whitehead writing *Principia Mathematica*, taking 360 pages to prove the proposition '1 + 1 = 2.'[119] Russell engaged in this fruitless exercise because he wanted certainty in the kind of way in which people want religious faith, as he wrote in 'Reflections on my Eightieth Birthday' in 1952. Continuing, he said,

> I thought that certainty is more likely to be found in mathematics than elsewhere. But I discovered that many mathematical demonstrations, which my teachers wanted me to accept, were full of fallacies, and that, if certainty were indeed to be found in mathematics, it would be a new kind of mathematics, with more solid foundations than those that had hitherto been thought secure.[120]

It was all to no avail, for in 1931, Gödel proved in 'On Formally Undecidable Propositions of *Principia Mathematica* and Related Systems I' that paradoxes cannot be eliminated from the foundations of mathematics.[121] This is not surprising for the relativistic world of form is essentially dual and dualistic. So if we reject paradoxes from our reasoning, the cognitive maps or conceptual models that guide our lives will not be a valid representation of the world we live in and will lead us dangerously astray.

We can free ourselves of the delusions that arise from such faulty reasoning by using Integral Relational Logic as the Cosmic Foundation and coordinating framework for all our learning. By transcending the categories and unifying all opposites in order to form the concept of the Transfinite Absolute, we can embrace paradoxes and self-contradictions with the Cosmic Equation, where *A* is any being, *W* is any whole, ∪ is union, and ~ is not:

$$W = A \cup {\sim}A$$



This universal, irrefutable truth is the keystone of the Universe, its fundamental design principle, enabling us to unify the incompatible concepts of God and Universe in Wholeness, as illustrated in this diagram. Uncertainties that are inevitable in an ever-changing world disappear for we then know God with Absolute Certainty. This is absolutely essential if we are to reveal Love and Peace, which are ever present within us, and thereby participate in cocreating a world in which we all live in love, peace, and harmony with each other and our environment.

This is quite possible, for this line of reasoning shows that the Truth that sets us free is present in the very foundations of mathematics. The Cosmic Equation is a theorem in mathematical logic that cannot be proven to be true from any set of axioms. As such, it is the equation that Albert Einstein spent the last thirty years of his life seeking to find at the heart of his unified field theory.[122] It is the simple, elegant, all-encompassing equation that can explain everything, the key that opens all the innermost secrets of the Universe. For the Cosmic Equation is the equation that Stephen W. Hawking has spent a lifetime searching for, as we were told in the movie *The Theory of Everything*, receiving an Oscar for Best Actor for Eddie Redmayne in 2015.

Nothing is more important at these times of unprecedented crisis. The Transfinite Great Attractor for all our lives is the Divine, as the union of Wholeness and Oneness, which is ever present as Ultimate Reality, but which can nevertheless be reached through evolutionary and involutionary introspective spiritual practices, as *The Four Spheres* explains.

Population growth

Such practices are absolutely essential if we are come to terms with the inevitable death of *Homo sapiens* within the next few generations, using the wonderful opportunity that evolution is giving us as we pass through its Accumulation Point on the way to it Glorious Culmination.

To see why this is so, in this section we look at the way that mathematicians model the growth of populations, whether human or otherwise, setting this into evolution as a whole in the following section. For the functions and equations that map population growth are applicable in a host of different dynamic systems, as Robert C. Hilborn and Steven H. Strogatz explains in their excellent books *Chaos and Nonlinear Dynamics* and *Nonlinear Dynamics and Chaos*, respectively

In this section, we look at just three examples. First Fibonacci, who studied the growth of a hypothetical colony of rabbits[123] in the thirteenth century. Secondly, in the nineteenth century, Pierre François Verhulst studied the population of Belgium, inspired to do so by *An Essay on the Principle of Population* by Thomas R. Malthus. And finally, we look at Robert May's twentieth-century studies of shoals of fish in a pond, which build on Verhulst's logistic model, within the context of chaos theory.

Fibonacci and the Golden Ratio

Before we look at the mathematics behind the growth of human population, we can look at the growth of a hypothetical rabbit population. In 1202, Leonardo of Pisa, known as Fibonacci (c. 1170–c. 1250), published a book titled *Liber Abaci* (*Book of Calculation*), which popularized the Hindu–Arabic numeral system in the Western World. As an example of this numerical notation, familiar to us today, Fibonacci considered the growth of an idealized (biologically unrealistic) rabbit population in which a pair of rabbits mate at one month old, the female giving birth to another pair of rabbits after a one month gestation

period. They immediately mate again, giving birth to another pair one month later, a reproductive process that continues indefinitely.

So, at time 0, a newly born pair of rabbits, one male, one female, are put in a field. At time 1, they mate, but there is still only one pair. Then, at time 2, there are two pairs in the field. The siblings born at this time mate with each other at time 3, producing a new pair at time 4, a process that continues with every pair that is born. At time 3, there are thus three pairs, at time 4, five pairs, and so on, for each pair who are born produce a new pair two months later. This birth rate gives rise to this series of numbers, which Édouard Lucas (1842–1891) over 600 years later called the 'Fibonacci sequence':[124]

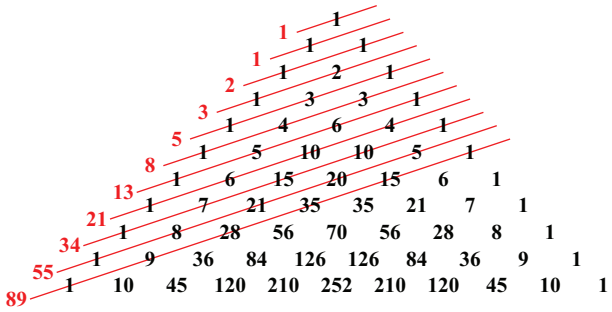
1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, …

In algebraic terms, we have, as an example of a linear recurrence equation:

$$F_n = F_{n-1} + F_{n-2}$$

where the seed values are either $F_1 = 1$ and $F_2 = 1$ or $F_0 = 0$ and $F_1 = 1$.

Not surprisingly, as the Fibonacci numbers are formed by successively adding previous numbers together in an accumulative manner, there is a relationship between this sequence and the binomial coefficients in the Pascal triangle. They form the sum of 'shallow' diagonals in the triangle, illustrated here. In algebraic terms,



$$F_n = \sum_{k=0}^{\left\lfloor \frac{n-1}{2} \right\rfloor} \binom{n-k-1}{k}$$

where $\lfloor m \rfloor$ means round up to the next highest integer.

Now just as a geometric progression is a series of exponential terms in the form r^n, we might well ask what the value of r might be, as Kepler did in 1611 in *The Six-Cornered Snowflake*, concluding, "It is impossible to provide a perfect example in round numbers."[125] For what we need is a value of r that satisfies the Fibonacci sequence such that

$$r^n = r^{n-1} + r^{n-2}$$

Well, dividing this equation by r^{n-2}, we have

$$r^2 - r - 1 = 0$$

This is a quadratic equation whose roots are

$$\frac{1 \pm \sqrt{5}}{2}$$

The positive root is the Golden Ratio, generally denoted by the Greek letter *phi*, φ, although when I was at university in the 1960s we used *tau*, τ. The negative solution is 1 - φ, often called *psi*, or ψ. In other words, if r is either φ or ψ, then r^n satisfies the Fibonacci relation. Let us now form another series, such that

$$U_n = a\phi^n + b\psi^n$$

This also satisfies the Fibonacci relation, for

$$U_n = U_{n-1} + U_{n-2}$$

If this relation is also to satisfy the Fibonacci series for F_1 and F_2 equal to one, we find that a and b are $1/\sqrt{5}$ and $-1/\sqrt{5}$, respectively. So, as $\sqrt{5} = \phi - \psi$, the nth term in the Fibonacci series is given as

$$F_n = \frac{\phi^n - \psi^n}{\phi - \psi}$$

Writing this out in full, we have this monstrous expression. No wonder that Kepler did not find it.

$$F_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$$

This formula is usually credited to the French mathematician Jacques Philippe Marie Binet (1786–1856), although it was well understood a century earlier by Bernouilli and others.[126] It is a rather strange formula, for F_n is an integer, and the right-hand side does not appear to be so. However, the root-fives cancel out for all values of n. So all is well.

As an aside, when Lucas was studying the patterns in the Fibonacci series in the 1800s, he formed what is now called a Lucas series with some interesting properties, thus:

$$L_n = F_{n-1} + F_{n+1}$$

where $F_0 = 0$. This gives this simple formula for the nth Lucas number:

$$L_n = \phi^n + \psi^n$$

The Lucas numbers are:

1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, …

These numbers have many surprising properties, one of which is a gem. If n is a prime number, then the number $L_n - 1$ is divisible by n. For instance, $L_7 = 29$, and 28 is divisible by 7. In other words, if n is prime, then

$$L_n - 1 = cn$$

For instance, for n = 2, 3, 5, 7, 11, 13, 17, and 19 c is 1, 1, 2, 4, 18, 40, 210, and 492. This relationship affects the very heart of mathematics because it shows an unlikely connection between a linear recurrence equation and the prime numbers, those that have no factors other than themselves and one. For primes are the basic building blocks in number theory, at the heart of the 'fundamental theorem of arithmetic', proved by Euclid: "Every integer n greater than 1 can be factored into a product of primes in only one way".[127]

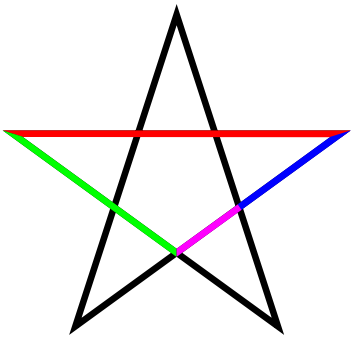
We now need to consider what happens to F_n as n gets larger and larger. To do this, we first note that φ or ψ are approximately 1.618034 and -0.618034, respectively. This means that the second term in the formula tends to zero as n approaches infinity. So, for large n, F_n is approximately

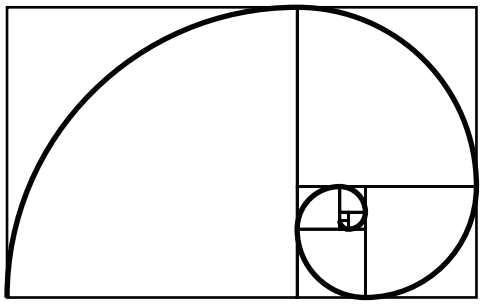
$$F_n = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n$$

We can thus see that the limit of the ratio of successive terms in the Fibonacci series tends to the Golden Ratio, rather like a geometric series where r converges on a limit, rather than remaining constant. We see another example on page 46, in the convergence of the Feigenbaum constant, which we use to consciously pass through evolution's Accumulation Point. So we have

$$\lim_{n\to\infty} \frac{F_{n+1}}{F_n} = \frac{1+\sqrt{5}}{2} = \phi$$

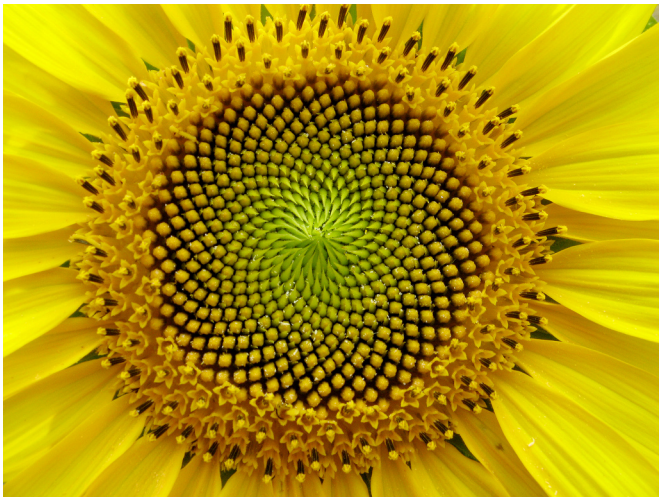
Now, as many people know today, especially those interested in sacred geometry, the Golden Ratio appears frequently in both geometry and nature. For instance, the coloured lines in this pentagram are all in the ratio of φ. In three dimensions, the Golden Ratio appears in many polyhedra with dodecahedral and icosahedral symmetries, such as the small and great stellated dodecahedra, discovered by Johannes Kepler in 1619 in his beautiful book *Harmony of the World*.





Regarding, the way that the Golden Ratio plays a central role in growth processes, here is a diagram of the Fibonacci spiral, which is an approximation to the Golden spiral, with the arc in each square being a quarter of a circle. Here, the length of the succeeding sides of each square in the diagram is given by the Fibonacci series: 1, 1, 2, 3, 5, 8, 13, 21, and 34.

Such spirals also occur in nature. For instance, here is a picture of a sunflower, showing how the seeds spiral out from the centre of the flower. As the Golden Ratio arises from the growth of linear recurrence equations in mathematics, it is not surprising that we see it frequently in nature. For instance, here is a list of examples I have found on the Web.[128]



Flower petals: The number of petals on some flowers follows the Fibonacci sequence. It is believed that in the Darwinian processes, each petal is placed to allow for the best possible exposure to sunlight and other factors.

Seed heads: The seeds of a flower are often produced at the center and migrate outward to fill the space. For example, sunflowers follow this pattern.

Pinecones: The spiral pattern of the seed pods spiral upward in opposite directions. The number of steps the spirals take tend to match Fibonacci numbers.

Tree branches: The way tree branches form or split is an example of the Fibonacci sequence. Root systems and algae exhibit this formation pattern.

Shells: Many shells, including snail shells and nautilus shells, are perfect examples of the Golden spiral.

Spiral galaxies: The Milky Way has a number of spiral arms, each of which has a logarithmic spiral of roughly 12 degrees. The shape of the spiral is identical to the Golden spiral, and the Golden rectangle can be drawn over any spiral galaxy.

Hurricanes: Much like shells, hurricanes often display the Golden spiral.

Fingers: The length of our fingers, each section from the tip of the base to the wrist is larger than the preceding one by roughly the ratio of *phi*.

Animal bodies: The measurement of the human navel to the floor and the top of the head to the navel is the Golden ratio. But we are not the only examples of the Golden ratio in the animal kingdom; dolphins, starfish, sand dollars, sea urchins, ants and honeybees also exhibit the proportion.

DNA molecules: A DNA molecule measures 34 angstroms by 21 angstroms at each full cycle of the double helix spiral. In the Fibonacci series, 34 and 21 are successive numbers.

Pierre François Verhulst and the logistic curve

However, Fibonacci's assumption that populations can growth indefinitely is clearly false. Populations of any breeding species, humans or otherwise, are intrinsically limited by the physical environments in which they live. But how can such constraints be represented in mathematical formulae? Well, the only-recently known mathematician Pierre-François Verhulst (1804–1849) was the first to give this question serious consideration. He worked at the Royal Military Academy of Belgium in Brussels and tried to predict the demographic evolution of the young Belgian society, formed as a constitutional monarchy in 1831.[129]

To celebrate the two hundredth anniversary of Verhulst's birth, acknowledging his critical contribution to the history of ideas, a group of scientists held a conference in September 2004 at the Royal Military Academy under the patronage of the King of the Belgians and sponsorship of the European Union. They published some of the papers presented there in *The Logistic Map and the Route to Chaos*, showing how

Verhulst's formula is related to chaos theory, fractals, the Fibonacci series, and nonlinear and statistical physics.

Regarding population growth, Verhulst wondered what would be the maximum sustainable population of Belgium given its limited resources? He presented the first results of his inquiries in 1844 to *Mémoires de l' Académie*, published a year later as '*Recherches mathématiques sur la loi d'accroissement de la population*' ('Mathematical investigations of the law of population growth').[130]

To see how Verhulst developed his mathematical model, let us first suppose that population growth is not constrained, that it progresses at a constant growth rate r during the whole lifetime of the process, and that this growth rate is independent of the current population level at time t. That is, when dealing with human population growth, Malthus suggested that the growth speed of a population is proportional to the current population level. When no other external constraints on the rate of growth are considered, then:

$$\frac{dp}{dt} = rp$$

where p denotes the population level at time t. The solution of this differential equation is an exponential function,

$$p(t) = p_0 e^{rt}$$

where p_0 is the population at time zero. For, as we see on page 10, the exponential function has the key characteristic that its derivative is the function itself, in this case multiplied by a constant r.

We now need to consider how to impose constraints on growth in a mathematical formula as the population approaches a maximum m. For instance, we could assume that natality (birth rate) and mortality rate decrease and increase in some form of relationship to population density, reaching equality at the maximum population.

For instance, if r is the difference between the birth and death rate ($B - D$), we could say that the birth and death rates decrease and increase in proportion to the level of the population at any one time, giving these simple relationships $B = b' - bp$ and $D = d' + dp$. So we have a revised formula for the growth rate, $r = (b' - d') - (b + d)p$ or $r = g - hp$, let us say. Plugging this into the Malthusian formula for unlimited population growth gives

$$\frac{dp}{dt} = gp - hp^2$$

This is a differential equation whose solution is[131]

$$p(t) = \frac{gp_0}{hp_0 + (g - hp_0)e^{-gt}}$$

where p_0 is the population at time zero. This is a maximum when t is infinite. So

$$m = \frac{g}{h}$$

As g is the difference between the basic birth and death rates, we can replace g with r and divide by hp_0, giving what Verhulst called the logistic function:

$$p(t) = \frac{m}{1 + ce^{-rt}}$$

where

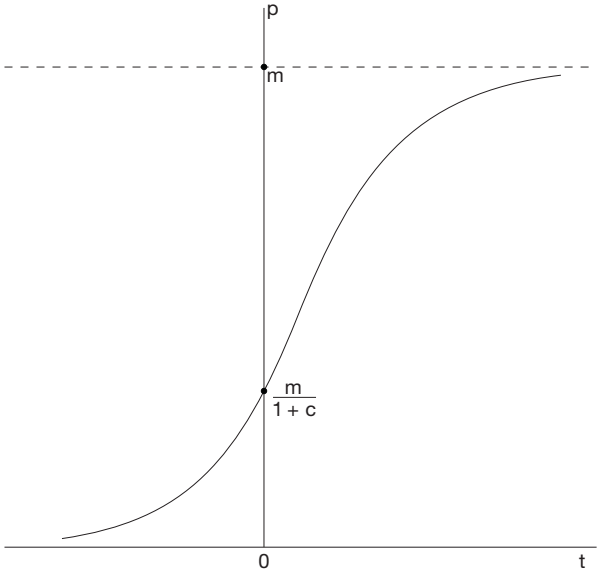
$$c = \frac{m - p_0}{p_0} = \frac{m}{p_0} - 1$$

The original differential equation then becomes

$$\frac{dp}{dt} = rp\left(1 - \frac{p}{m}\right)$$

When I first came across the logistic function in *The Penguin Dictionary of Economics* in the early 1980s, it was not clear to me why it is so named, a situation that has also puzzled mathematicians. Nevertheless, Hugo Pastijn suggests that the logistic function is so called because it is based on Greek *logisticos* 'art of computation', perhaps one meaning of the French word *logistique* at Verhulst's time.[132]

It is a rather curious function, for as Daniel M. Dubois points out, "This means that the system loses its initial value and that its future is completely defined by the value of the parameter *m*, the value of which being fixed. So, such an equation is an anticipative system in the sense that the final future value, *m*, of the population is already known and completely fixed at the present time."[133]



Here is a chart of the logistic function, showing the characteristic S-shape of the growth curve, asymptotically approaching *m* as *t* tends to infinity and to zero backwards in time to minus infinity. The growth curve, often called the learning curve in psychology, is ubiquitous, appearing in a multitude of fields, including these, listed in Wikipedia: artificial neural networks, biology (especially ecology), biomathematics, chemistry, demography, economics, geoscience, mathematical psychology, probability, sociology, political science, and statistics.

Its economic use is simply illustrated. When a new product is introduced into the marketplace, sales sometimes grow rather slowly at first until the product 'takes off'. There is then a period of rapid growth until the product reaches its saturation point. Such a saturation point is often determined by the finite population or number of households, where one refrigerator or vacuum cleaner, for instance, is quite enough.[134]

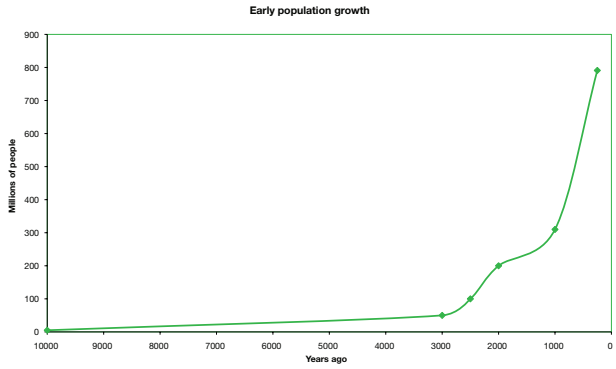
However, this chart can appear a little misleading, for the population at time zero is far from the beginning of the curve, which is actually minus infinity. We can rectify this by changing the coordinate system, setting $t = t' - d$, where d is a suitable finite number where p is sufficiently close to zero. The population at time zero is then

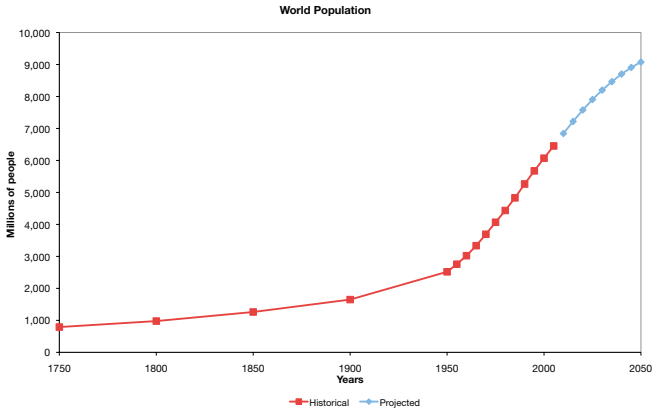
$$\frac{m}{1 + ce^{rd}}$$

This is the pure mathematics, but it is not always clear what the parameters need to be when applying the function to actual situations. Verhulst, himself, compared "the real population figures of France, Belgium, Essex and Russia with the result of his calculations. The correspondence was striking, although the available figures related to a period of only twenty years."[135] In hindsight, the situation is rather different. "With the model Verhulst introduced in 1844 he predicted that the maximum size of the Belgian population would be six million and six hundred thousand individuals. Presently [2006] Belgium has a population of roughly eleven million."[136]

Regarding the global human population, growth is depicted in the two charts below, using figures from 2010. The first is from 10,000 years ago, when our forebears stopped being hunter-gatherers to settle in villages, to 265 years ago, at the birth of the industrial revolution. The second chart shows what has

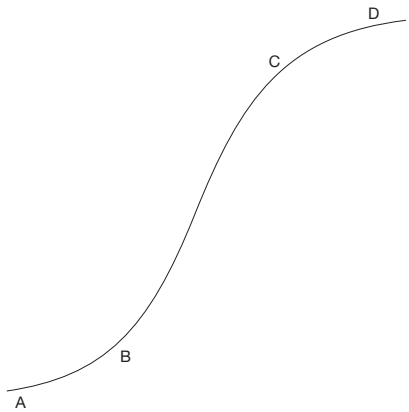
been happening since then projected to 2050.[137] These two graphs show the characteristic S-shape of the growth curve, with a very slow beginning, in this case, which is today beginning to reach its saturation point, as projections of population growth level off.



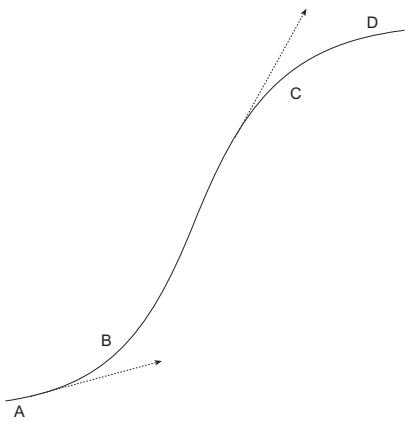


In biology, the seminal work on the study of growth processes is D'Arcy Wentworth Thompson's *On Growth and Form*, first published in 1917, revised in two volumes in 1942. Thompson made much use of Verhulst's logistic curve in his 208-page chapter 'Rate of Growth', illustrating his brilliant skills as a mathematical biologist, pointing out that this one curve recurs in endless shapes and circumstances, for mathematics generalizes and "is fond of giving the same name to different things".[138] For instance, he pointed out that it appears in hysteresis, where the value of a physical property lags behind changes in the effect causing it, as for instance when magnetic induction lags behind the magnetizing force. Yet, you will learn very little about the rate of growth in John Tyler Bonner's 1961 abridgement of *On Growth and Form* for the subject of growth is almost completely ignored,[139] perhaps because of the mathematics involved.

Perhaps these difficulties of applying the logistic function in actual situations is one reason why there is no mention of Verhulst in any of my four books on the history of mathematics. But this does not mean that we should ignore it, for the sigmoidal shape can intuitively help us to understand growth processes, wherever they may occur. For instance, C. H. Waddington called the growth curve a 'tool of thought',[140] and Stephen Jay Gould[141] and Peter Russell,[142] for instance, have followed suit.



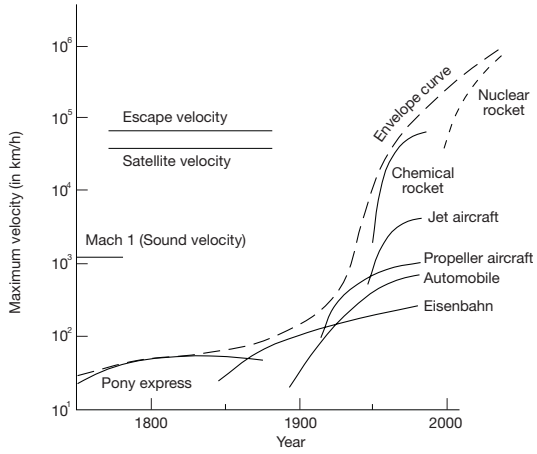
In the general populace, the growth curve is generally called the learning curve, familiar to many. For we often hear people saying that a particular subject has a 'steep learning curve'. However, what they are generally referring to here is the flat part at the beginning of the curve, from A to B. When beginning a new project, it is easy to give up, saying, "I'll never manage this." However, eventually, we learn to coordinate the necessary skills and ideas at B, the coordination point, and learning progresses at exponential rates of development. But structures do not continue evolving indefinitely. Towards the end of any growth process, they reach a plateau at point C, the saturation point.



It is vitally important to understand the full shape of the growth curve, not extrapolating the various rates of change at different stages, depicted here. Most significantly, when growth is happening very fast, we might think that it can continue indefinitely, such as the deluded

belief that technological development can drive economic growth indefinitely in today's capitalist society. Gordon E. Moore, the initiator of Moore's law and cofounder of Intel, is well aware of the limits of evolutionary growth. As he told a meeting of the world's top chip designers and engineers on 10th February 2003, "No exponential is forever." Irrationally, he then went on to say, "Your job is to delay forever."[143]

Our learning often results in new technologies, which can extend learning curves when one particular technology reaches its saturation point. For instance, our mode of transport has been getting faster and faster since the beginning of the industrial revolution, when we no longer relied on just animal power to move around the Earth, whether our own power or that of horses and other animals. This situation is well illustrated in this diagram, which shows how a growth curve can be depicted as an envelope of a set of growth curves (the Eisenbahn is a railway line in Germany as far as I have been able to ascertain).[144] However, this process does not continue indefinitely. On 24th October 2003, Concorde made its last commercial flight accompanied by eloquent outpourings from the journalists. This is a clear indication of the slowing down of technological development.



But does this mean that technological development cannot drive economic growth indefinitely? Well, this example is but one of many in which we humans have invented tools to extend our rather limited physical abilities over the years, others being the flint axe, wheel, printing press, telescope, steam engine, and telephone. However, the stored-program computer, invented in the late 1940s, is a quite different type of machine. The computer is a tool of thought, able to extend the human mind, even in some cases replacing it.

Believing that humans are machines and nothing but machines, this has led some computer scientists to predict that they will shortly create machines with artificial general intelligence, far exceeding any level of intelligence that we humans have the potential to develop. If this were true, it would clearly have devastating economic consequences, as the cycle between humans as both producers and consumers of goods and services would be broken. However, such an assumption is not true. There are actually no limits for human cognitive potential, as we can see when Life heals our fragmented, split minds by teaching us how to integrate all knowledge in all cultures and disciplines from all times into a coherent whole. For that whole is Ineffable, Nondual Wholeness, evolution's Glorious Culmination, which transcends all the categories, enabling us to see the Totality of Existence from a Holoramic 'Whole-seeing' perspective, rather than our narrow ego- and anthropocentric points of view.

Standing outside the narrow timeframes of our lifespans as individuals and as a species helps us to put another vital issue into perspective: our dependence on fossil fuels, such as coal, oil, and gas. This is what M. King Hubbert, Chief Consultant (General Geology) for Shell, did in 1956, when writing the seminal 56-page paper on what is today called 'peak oil', extending it into a 150-page report on 'Energy Resources' in 1962.

In the first report, titled 'Nuclear Energy and the Fossil Fuels', Hubbert pointed out that the fossil fuels that we have been rapidly consuming over the centuries and millennia, especially since the beginning of the industrial revolution, originated "from plants and animals existing upon the earth during the last 500 million years. The energy content of these materials bas been derived from that of the contemporary sunshine, a part of which has been synthesized by the plants and stored as chemical energy."[145]

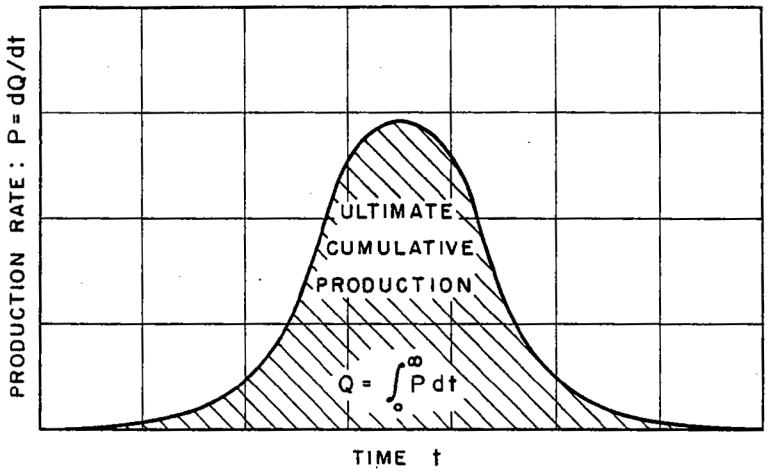
Continuing, Hubbert pointed out,

> Throughout all human history until about the thirteenth century, the human race, in common with all other members of the plant and animal complex, had been solely dependent upon the contemporary solar energy which it had been able to command. This comprised the energy from the food it was able to consume, that of the wood burned for fuel, and a trivial amount of power obtained from beasts of burden, from wind, and from flowing water.[146]

However, this situation began to change "when the inhabitants of northeast England discovered that certain black rocks found along the seashore, and thereafter known as 'sea coles', would burn. Thus began the mining of coal and the first systematic exploitation of the earth's supply of fossil fuels." However, such exploitation cannot continue indefinitely, which became crystal clear in December 2015, when the last coalmine in the UK closed, bringing an 800-year industry to an end, in that country, at least. This is a clear indication that vital fossil fuels are a finite resource, which cannot be consumed indefinitely.

Accordingly, Hubert set out to develop a mathematical model of the production rate of fossil fuels so that they could be better understood. He began with some statistics, plotting the world production of coal in millions of metric tons per year from 1820 to 1960, the world production of crude oil in millions of barrels per year from 1860 to 1960, and the annual production of coal, crude oil, and gas in the USA. He then noticed a 'strong family resemblance' among the eight charts he used as examples: "Each curve starts slowly and then rises more steeply until finally an inflection point is reached after which it becomes concave downward."[147]

In the 1956 report, Hubbert made no attempt to express this pattern in an explicit mathematical formula. All he did was to derive a production curve from the curves for production rate. He noted "for any production curve of a finite resource, … the production rate will be zero when the reference time is zero, and the rate will again be zero when the resource is exhausted." To show the relationship between the production and the cumulative production curves, he produced this diagram, captioned 'Mathematical relations involved in the complete cycle of production of any exhaustible resource', illustrating the familiar bell shape of growth and decay of any finite resource, as an example of all life-and-death processes in the Universe.[148]



In mathematical notation, if $P(t)$ and $Q(t)$ are functions denoting the production and cumulative production of a finite resource, they are the inverse of each other, by the fundamental theorem of the calculus, introduced on page 9. We thus have these abstract formulae, which are the key to understanding the mathematics of growth and decay processes.

$$P(t) = \frac{dQ}{dt}$$

and

$$Q(t) = \int_0^s P dt$$

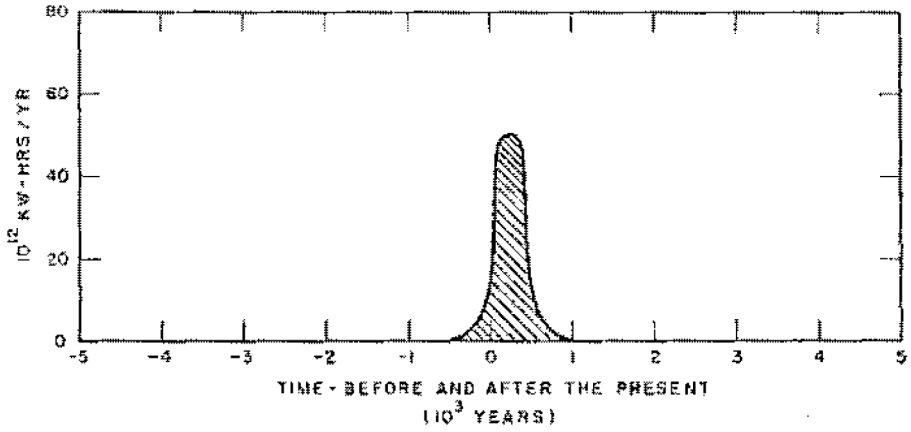
When $s = \infty$, we have

$$Q_{max} = \int_0^{\infty} P dt$$

Then, in the 1962 report, Hubbert used the logistic growth curve as $Q(t)$, pointing out that growth phenomena start slowly, gradually accelerate, and finally level off to a maximum, describable by an empirical equation of the form:[149]

$$Q(t) = \frac{Q_{max}}{1 + ce^{-rt}}$$

There is no need to go into any further detail into how mathematicians develop these formulae and apply them in actual situations, for the purpose of this essay is to see the essential patterns in growth and decay processes, describing the basic mathematics underlying them. Also, if we go into too much detail, we can lose touch with the broader perspective. To help us see this, in his second report, Hubbert showed how the exploitation of fossil fuels could be seen in a span of history extending for some thousands of years before and after the present, presenting this diagram.[150]



We can see here that humanity's exploitation of fossil fuels is but a brief episode in the overall scale of things. The significant timespan in which cumulative production increases from, say, 10 per cent to 90 per cent of the ultimate reserves is really quite tiny. At the time, Hubbert estimated that the figure for coal would be about 350 years, estimating that 80 per cent of petroleum reserves will be produced between 1960 and 2040. Given the additional information we have today, perhaps more accurate estimates could now be made.

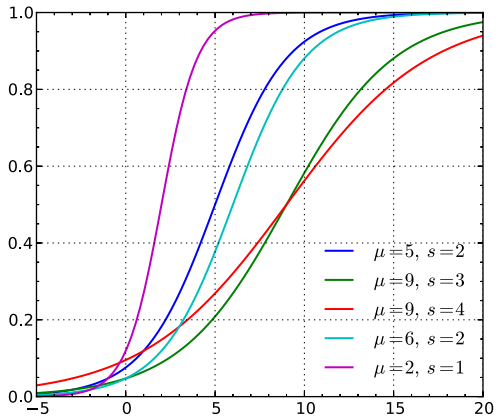
However, whatever these estimates might be, Alfred J. Cavallo pointed out in the opening words of a short article on Hubbert's 'Petroleum Production Model' 2004 in "Although it is understood nominally that petroleum and natural gas are not renewable resources, modern industrial society has been operating successfully as if they were," going on to say, "It is clear that the concept of limits to growth is difficult for most people to accept. Many decades of increased economic activity and spectacular progress in all areas of science and technology have created the impression that such a state of affairs is a permanent component of Western civilization. Yet limitations are a fundamental part of the natural order, and sooner or later we must face up to this."[151]

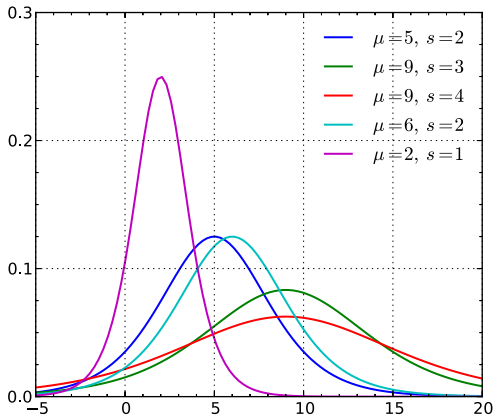
By adjusting the constants in the logistic function, it can be interpreted as a cumulative distribution function in probability theory. Its derivative then becomes a probability distribution function, not unlike the more familiar bell shape of the normal distribution function in statistics.

Here is the equation for the cumulative logistic distribution, where μ is the mean and s is the standard deviation:

$$F(x) = \frac{1}{1 + e^{-\frac{x-\mu}{s}}}$$

Here m = 1, for probabilities lie between zero and one, illustrated on the right in the family of curves for various values of μ and s. And to the left is the corresponding set of curves for the logistic probability distribution function.





Such functions are ubiquitous in all the sciences, including physics, where they are used to calculate the probabilities of the positions of electrons in electron clouds. I have written more on probability theory in other publications available on my website. However, in order to focus attention on the growth and decline of human population, this is as much as I want to say on the subject in this version of this essay.

Robert May and chaos theory

Now, while there are many ways of dampening population growth in differential equations, other than the basic logistic function, such functions are not necessarily the best way of representing growth processes in mathematics. As Hugo Pastijn points out, "This continuous time model is finally not as universally valid as it was sometimes considered. In addition, it is now recognized that the continuous time model does not always reflect reality. When there are for instance jump-wise simultaneous behavioural changes of all the individuals of the population, the structural dynamics of the population may fundamentally change."[152]

To represent such behavioural changes in mathematics, there is one other vitally important technique to which we must now turn our attention. We need to transform continuous differential equations into difference equations, their discrete counterparts. In this essay, we only need to consider the basic logistic function, whose differential equation, to remind you, is:

$$\frac{dp}{dt} = rp\left(1 - \frac{p}{m}\right)$$

To convert this differential equation into its discrete form, mathematicians find it convenient to normalize the carrying capacity to 1.[153] So the equation becomes:

$$\frac{dp}{dt} = rp(1 - p)$$

We can then convert this into the difference logistic equation for discrete variables, using a technique devised by Euler:[154]

$$p(t + \Delta t) - p(t) = \Delta t(rp(t) - rp^2(t))$$

where Δt is a discrete increment of time. As it is convenient to take Δt as one, without any loss of generality, so that the units of time are days, years, or whatever, we have, writing the equation in its more convenient discrete form:

$$p_{n+1} = (1 + r)p_n - rp_n^2$$

If we now set $a = 1 + r$, so that a growth rate of 5% become 105%, for instance, and

$$x_n = \frac{p_n}{1 + \frac{1}{r}}$$

we obtain the canonical form for the difference equation for the logistic function,[155] known as the logistic map,[156] where the term 1 - x_n keeps the growth within bounds, since as x_n rises, 1 - x_n falls.[157]

$$x_{n+1} = ax_n(1 - x_n)$$

This difference equation is a first-order one, unlike the Fibonacci function, which is second-order, each term being a function of two preceding ones, rather then just one in the logistic function. However, this is nonlinear, like the quadratic recurrence relation for the Julia and Mandelbrot sets on page 17, but unlike the linear Fibonacci function on page 32. This has led to some quite extraordinary results, amazing mathematicians when they came to study them in the 1970s.

While many mathematicians were involved in this evolutionary process in the noosphere, Robert M. May wrote a seminal paper on first-order, nonlinear difference equations in 1976, when on leave from the Biology Department, Princeton University at King's College Research Centre, Cambridge. Australian-born May, now Baron May of Oxford, later became Chief Scientific Adviser to the UK Government and president of the Royal Society of London from 2000 to 2005.

His introductory paper is titled, 'Simple Mathematical Models with Very Complicated Dynamics', highlighting the central importance of this quite new field in mathematics. For as he said in his concluding remarks, the mathematical intuition developed from studying linear systems, which tend "to dominate even moderately advanced University courses in mathematics and theoretical physics," ill-prepares "the student to confront the bizarre behaviour exhibited by the simplest of discrete nonlinear systems", such as the logistic difference equation. He therefore urged students to be introduced to such equations early in their mathematical education.[158]

Well, this has obviously happened, for the nonlinear logistic difference equation is but one of many that lead, not to a single value or a finite set of numbers, but rather a random set, a situation that Tien-Yien Li and James A. Yorke called 'chaos' in 'Period Three Implies Chaos' in 1975.[159] They were inspired to write this paper by a classic one that Edward N. Lorenz had written in 1963 titled 'Deterministic Nonperiodic Flow'.[160]

Lorenz was originally a weather forecaster by profession and in the 1950s he "rapidly became sceptical that linear statistical methods, then very much in vogue, could be successfully applied to the long-range prediction of such a nonlinear system as weather".[161] Accordingly, he set out to find nonlinear functions that could be used in weather forecasting, publishing his initial results in his celebrated, seminal paper.

One result of his studies is that infinitesimal perturbations in a weather system could lead to cyclones and anticyclones in the atmosphere, an idea that was not universally accepted. As Lorenz said, "One meteorologist remarked that if the theory were correct, one flap of a sea gull's wings would be enough to alter the course of the weather forever. The controversy has not yet been settled, but the most recent evidence seems to favour the sea gulls." Later, suggestions from colleagues led Lorenz to use the more poetic *butterfly*, leading to the well-known butterfly effect.[162]

So in what way can mathematical techniques used in weather forecasting help us understand what is likely to happen to humanity in the next few decades? Well, one of the key concepts arising from Lorenz's studies is that of mathematical attractor, defined as 'A value or set of values toward which variables in a dynamical system tend to evolve'. *Attractor* has other meanings in physics, psychology, and in mysticism, and there is the popular law of attraction in the New Age movement. But is there something that all these meanings have in common?

Well, to keep things as simple as possible, the way that I approach this question is to go right back to first principles. We see on page 5 that a geometric series tends to infinity or a finite limit depending on whether r is greater or equal to one or not. But if $r = -1$, then

$$G_n = 1 - 1 + 1 - 1 + 1 - 1 + 1 - \cdots$$

In this case, G_n does not have a single value. It oscillates between 1 and 0, never resting in one or the other. Mathematical attractors in nonlinear difference equations have an even greater variety of behaviours. This is rather surprising, for the logistic function arising from its differential equation has a finite limit, denoting the maximum population that can be maintained in equilibrium.

In the case of the logistic difference equation, James Gleick tells us in his fascinating history of the evolution of chaos theory that May was studying it in the context of a hypothetical population of fish living in a pond, which, by its nature, is limited. But May was getting some very strange results. He first noted that the initial set of values of x_0—known as the basin of attraction—need to be in the range of 0 to 1, which enables all x to be in this range as the function is iterated. If $x > 1$, then x tends to -∞, which is not really extinction, as May calls it. Also, the maximum value that x can take is $a/4$, which means that the equation possesses non-trivial dynamical behaviour only if $a \leq 4$.

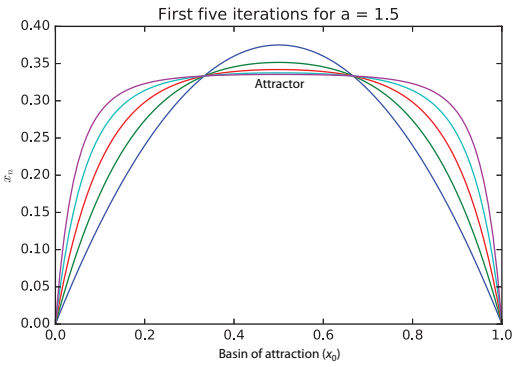
Regarding values $0 \leq a \leq 4$, if $a \leq 1$, x tends to 0, converging faster the closer a gets to 0. The population then becomes extinct, steadily diminishing year by year, for $a \leq 0$. Then for $1 < a \leq 3$, the attractor is a single value greater than zero. This happens when $x_{n+1} = x_n$ after a finite number of iterations. The value of x is then given by this equation:

$$x = ax(1 - x)$$

The convergent point c is then either zero or given in this expression:

$$c = \frac{a - 1}{a} = 1 - \frac{1}{a}$$

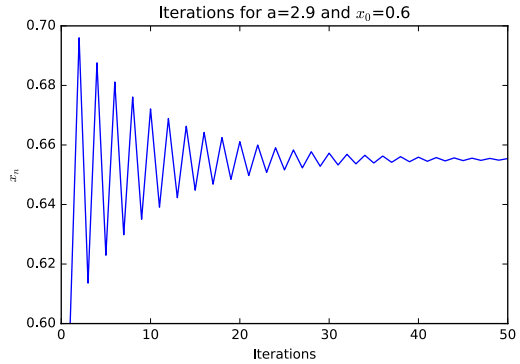
For instance, if a = 1, 1.5, 2, 2.5, and 3, c is 0, ⅓, ½, ⅗, and ⅔ or 0, 0.333, 0.5, 0.6, and 0.667. For instance, here are the first five iterations for a = 1.5, with all values of x_0, as the basin of attraction, converging on ⅓, as the attractor. The blue curve is a parabola, marking the first iteration for all valid values of a and x_0. If $x_0 = c$ or $1 - c$, the first iteration lands on the convergent point and no convergence is necessary. When $x_0 < c$, convergence increases monotonically towards c. And when $c < x_0 < 1 - c$, x_n decreases monotonically towards c. However, when $x_0 > 1 - c$, the values of x_1 correspond to those for $x_0 < c$, and convergence is not strictly monotonic, beginning to show signs of oscillations, becoming progressively obvious as a increases.

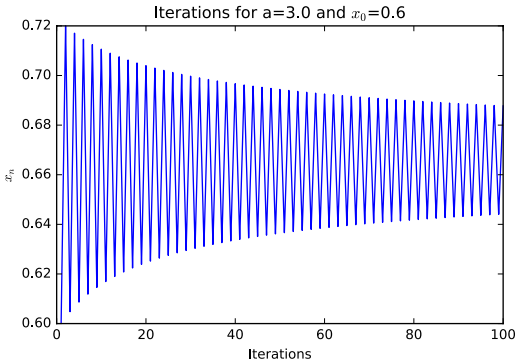


One important point about this diagram is that it does not matter what the initial value for x_0 is; the attractor is independent of this initial value, not unlike the way that the maximum population in the

logistic function is independent of the initial population. Another is that the closer that x_0 is to c, the faster the convergence on the attractor.

We notice this especially with $a > 2$, where convergence is more erratic, becoming oscillatory, like the left-hand diagram below, before converging on c. When a reaches 3, c is still mathematically a single point (⅔), but convergence is extremely slow as I discovered when my little IPython program using the Matplotlib class library went through one million iterations. The right-hand diagram shows just the first one hundred.





When $a > 3$ the attractor initially becomes two convergent points, rapidly bifurcating as a increases towards the point of accumulation for the function. This happens when $x_{n+2} = x_n$ after a finite number of iterations. The mathematics is a little trickier than for $x_{n+1} = x_n$ for we now have, dropping the suffixes,[163]

$$x = a^2x(1-x)(1-ax+ax^2)$$

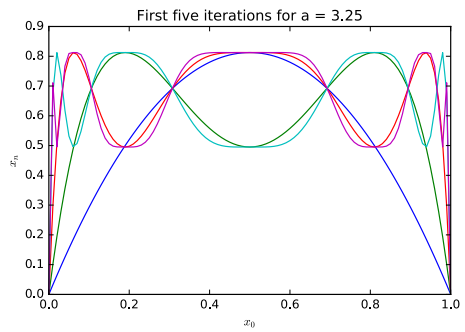
Rearranging the terms gives this quartic polynomial for x:

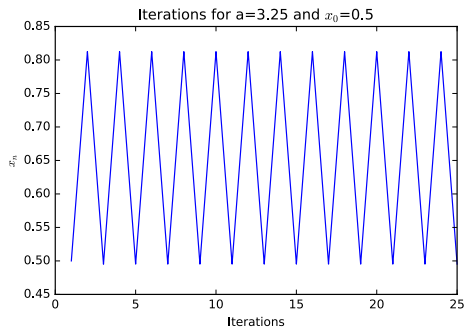
$$a^3x\left(x-\left(1-\frac{1}{a}\right)\right)\left(x^2-\left(1+\frac{1}{a}\right)x+(a+1)\right)=0$$

The quadratic factor has two roots for the convergent points c_1 and c_2 as the attractor:

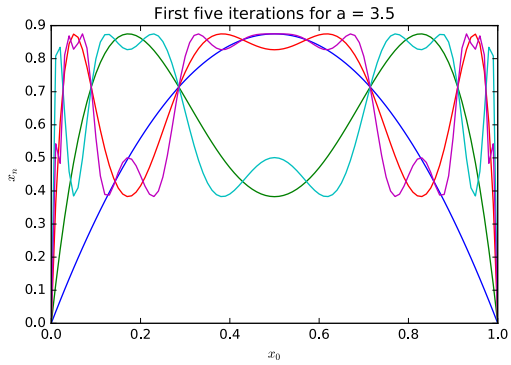
$$\frac{1}{2}\left(\left(1+\frac{1}{a}\right)\pm\frac{\sqrt{(a-3)(a+1)}}{a}\right)$$

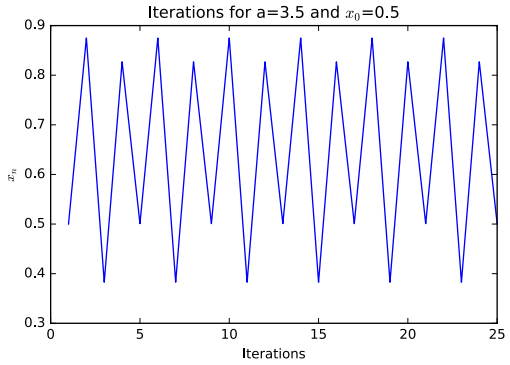
We can see from this expression that the roots are only real when $a \geq 3$, which is where bifurcations begin, with $c_1 = c_2$ when $a = 3$. For instance, the figure on the left shows the first five iterations for a = 3.25. Here, whether x is heading towards c_1 or c_2 in the second iteration (x_2) is dependent on the initial value of x_0. Subsequent iterations then flip-flop between the two attractor points, converging in complementary phases. How fast convergence happens varies considerably with different values of x_0 indicate. As the number of iterations increases, the connecting lines between $c_1 = c_2$ become steeper, although what happens when x_0 is at a crossover point is not clear. Presumably it takes an infinite number of iterations to converge on the attractor, if it does so at all. Be that as it may, the right-hand diagram well-illustrates the bifurcating values for $x_0 = 0.5$.





The mathematics becomes much more complex when 2-cycle oscillations break into four cycles, with c_1 and c_2 becoming c_{11}, c_{12}, c_{21}, and c_{22}. Mathematicians tell us that this happens when $a = 1 + \sqrt{6}$, which is about 3.449489. These 4-cycles are illustrated in these two diagrams for $a = 3.5$, with the oscillations moving between c_{11}, c_{22}, c_{12}, and c_{21} in sequence.





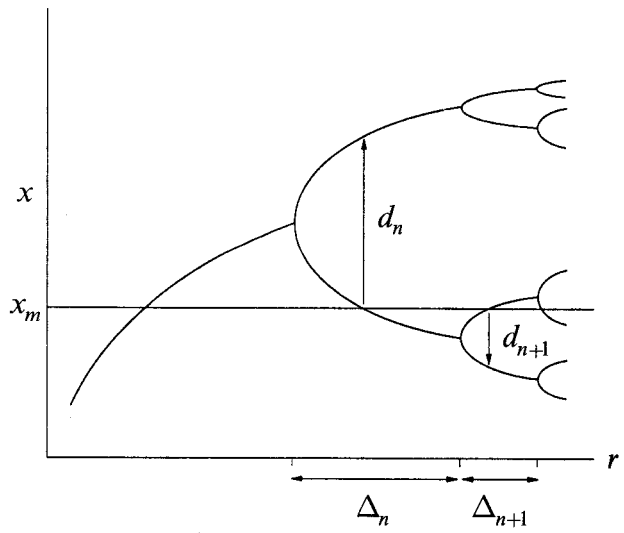
After this, bifurcations multiply exponentially faster and faster. To calculate the values of a_n for each of these points requires polynomials with degrees also accelerating exponentially, far beyond my abilities to calculate. Nevertheless, what we can see quite easily is that the bifurcation points converge on an accumulation point, rather like a geometric series when $r < 1$.

So what is going on here? Well, this was something that Mitchell J. Feigenbaum set out to discover in 1975, when studying the logistic map, inspired by Nicholas Metropolis, Myron Stein, Paul Stein, David Ruelle, and Stephen Smale, first publishing his results in 1978.[164] This was a rather slow process, because his "sole computing power resided in a programmable pocket calculator". Nevertheless this was an advantage, for he tried to anticipate the next value of λ_n, as he called a_n, and noticed that the values were converging geometrically to a value of 4.669. Paul Stein then reminded him that period doubling isn't a unique property of the quadratic map, but also occurs, for example, in this equation:[165]

$$x_{n+1} = \lambda \sin \pi x_n$$

So Feigenbaum set out on the even more laborious task of seeking the bifurcating points of this function, and to his amazement, he found exactly the same convergent point. He had found a mathematical constant in the depths of chaos theory, in cases where nonlinearity is not simply quadratic but is also transcendental, at the heart of what he called 'universality theory'. As he said, "This definite number must appear as a natural rate in oscillators, populations, fluids, and all systems exhibiting a period-doubling route to turbulence! … So long as a system possesses certain qualitative properties that enable it to undergo this route to complexity, its quantitative properties are determined."[166]

This diagram in Steven H. Strogatz's popular *Nonlinear Dynamics and Chaos* well illustrates the beginning of the progress towards the accumulation point, showing two universal constants that Feigenbaum discovered. The first does not need to concern us too much, for it is not applicable to evolution as a whole, as far as I can tell, which is the primary focus of this essay. Nevertheless, for completeness, d_n denotes the distance from x_m, the maximum value of the function, to the nearest point



in a 2^n cycle, giving α, the Feigenbaum reduction parameter, as:[167]

$$\alpha = \lim_{n\to\infty} \frac{d_n}{d_{n+1}} = -2.502907875\ldots$$

The Feigenbaum bifurcation velocity δ is given by this expression:

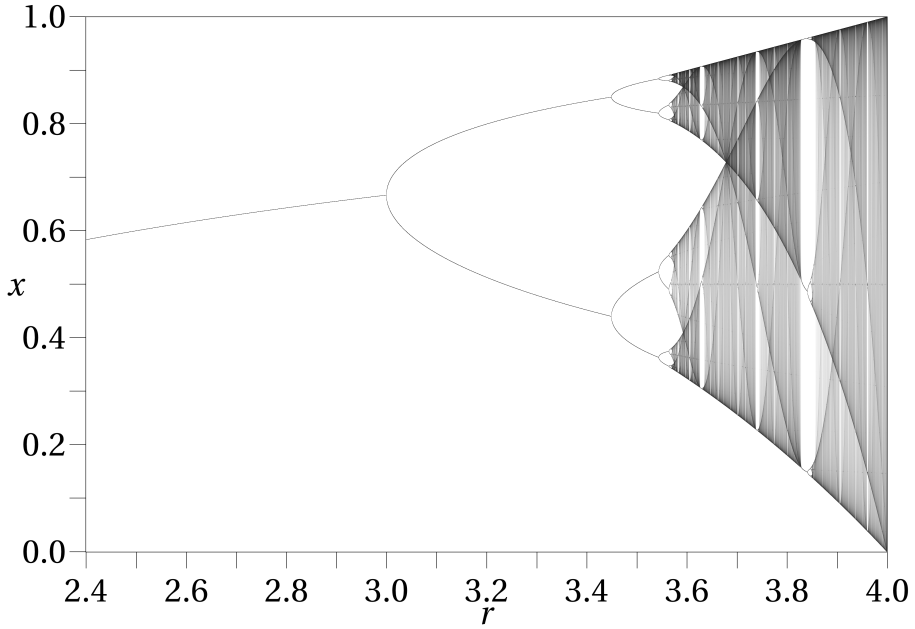
$$\delta = \lim_{n\to\infty} \frac{a_n - a_{n-1}}{a_{n+1} - a_n} = \frac{\Delta_n}{\Delta_{n+1}} = 4.669201609\ldots$$

What is amazing about these universal constants at the heart of chaos theory is that they don't appear to have any connection to any other fundamental mathematical constants, such as π, *e*, and φ. It is also not known if they are algebraic or transcendental. Indeed, how they can be calculated to hundreds of decimals places is still a bit of a mystery for me. They have appeared out of the blue in the fourth quarter of the twentieth century after hundreds and thousands of years of human thought, yet tell us so much about what is happening to humanity at the present time, as we look at in Subsection 'Evolution as a dynamical system' on page 60.

n	2^n	a_n	$1/r_n$
1	2	3	N/A
2	4	3.4494897	N/A
3	8	3.5440903	4.751447
4	16	3.5644073	4.656229
5	32	3.5687594	4.668321
6	64	3.5696916	4.668633
7	128	3.5698913	4.668002
8	256	3.5699340	4.676814
$\aleph_0$	$\aleph_1$	3.5699457	4.669202

One other point. δ is a limiting constant to which the ratio in the above limiting function converges, as this table illustrates, not monotonically, it seems. The accumulation point AP for the logistic map is 3.569945672 after a countable number of bifurcating points but with an uncountable number of values of x_n, yet contained within finite limits for each value of a > AP.

Nevertheless, complete chaos does not break out immediately. The infinite number of values of x_n are still contained within fairly narrow bands as this diagram, reproduced in much of the literature on the logistic map, illustrates. Here parameter r is a, May's original variable, to distinguish it from r in the continuous logistic function.

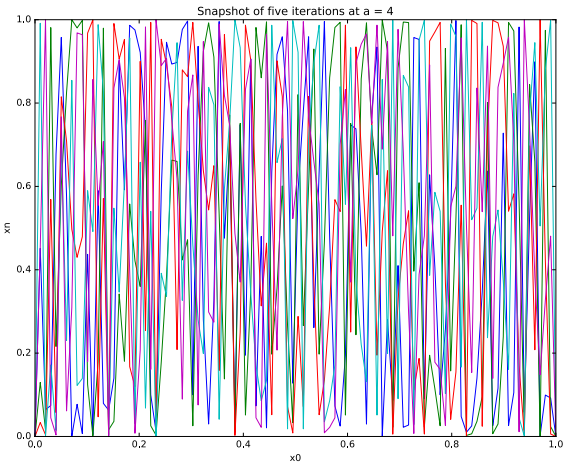


As can be seen in this diagram of the logistic map as a whole, patterns can still be seen as a moves towards its maximum 4. This is the macroscopic view. But as narrower and narrower ranges of a are studied with more and more precision, patterns of approximate self-similarity appear. For instance, a period-3 bifurcation occurs at $a = 1 + 2\sqrt{2}$ = 3.828427, which then bifurcates at 6, 12, and so-on intervals,

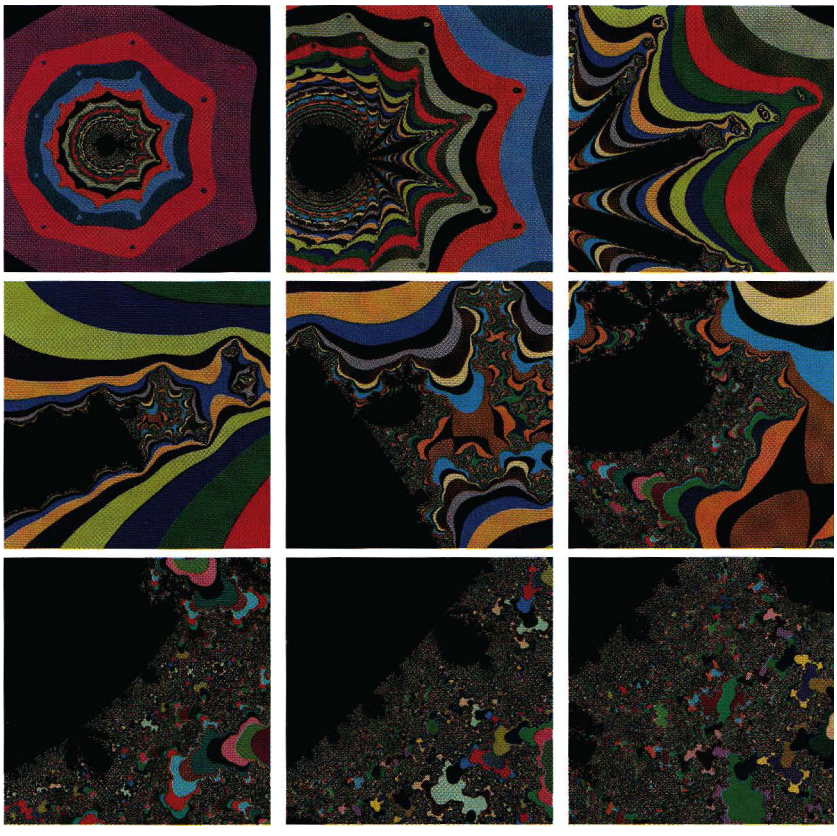
producing a tiny image of the entire map. This is rather like the way that the basic cardioid-circle structure appears in the depths of the Mandelbrot set, illustrated on page 20.

There is a degree of order in the chaos. How this deterministic chaos progresses is also extremely susceptible to a change in the initial conditions. Two sequences with almost identical values for x_0 will at first behave in a virtually identical manner, but then suddenly diverge so that from then on there is no correlation between the two oscillations. A similar sensitivity is also observed with respect to a change in the growth parameter a. Examples of this phenomenon are x_0 = 0,05000000000 and 0,05000000001 and a = 3,83000000000 and 3,83000000001.[168]

Yet, despite such patterns in the chaos, the logistic map can also be used as a random-number generator, when all values are equally possible. Perhaps this randomness only occurs at the limit, for here are iterations 96 to 100 for a = 4.



When a and x_0 are complex numbers, the logistic map can generate beautiful fractals, in a similar manner to the Julian and Mandelbrot sets. Values for a are calculated from an iteration sequence for a particular nonzero value of x_0. Like the other sets, when a-values are bounded, they are in the Verhulst fractal set and if not, points can be coloured depending on how fast they accelerate towards infinity. This picture illustrates the logistic fractal for the value $p_0 = -10^{-7}$. Each figure to the right and downwards is an enlargement of the preceding figure, showing that "the Verhulst set has a fractal structure, in the sense that however much we enlarge this boundary, it will never become a simple line or curve".[169]



Evolution's Accumulation Point

Having looked at the basic mathematics of the growth of populations, which are of such generality that they can be used to study a wide variety of other phenomena, we now need to look at how we can use these mathematical techniques to model evolution as a whole. For evolution is an accumulative process, accelerating faster and faster with each year that passes.

What is accelerating is not a material object, which requires a force to accelerate it, as Newton's famous equation $F = ma$ indicates. Rather, it is the growth of structure that is accelerating exponentially, whether structures be physical or nonphysical. And change is accelerating because structure, itself, is energetic, as David Bohm told me the first time I met him in 1980, six months after resigning from my marketing job with IBM.

So to understand what is causing us to behave as we do, we need to abandon the association of mass and energy, which Einstein encapsulated in his famous equation $E = mc^2$. Furthermore, structures are only alive and flourishing when they are constantly being fed with the creative power of Life, bubbling up from the Divine Origin of the Universe, like a fountain. This is not only crystal clear from human experience, it is also cognitively confirmed when we accept that the Cosmic Equation, defined on page 30, is the key that unlocks all the innermost secrets of the Universe.

Here is not the place to explore the integral science of causality that is embodied in the Unified Relationships Theory, for this is described in my other writings. All we need to note is that by the Principle of Unity, historically evolution has proceeded in bursts with long periods of comparatively little change in between. In 1971, Niles Eldredge and Stephen Jay Gould called this phenomenon 'punctuated equilibria', as an alternative to phyletic gradualism,[170] which held sway at the time.

As explained in *The Four Spheres*, one example of a sudden change in evolutionary history occurred at the birth of multicellular organisms in a process that Lynne Margulis made famous in the concept of *symbiogenesis*. In *Process and Reality*, when studying the concept of the Ultimate, Alfred North Whitehead called this convergent evolutionary process *concrescence*, from the Latin *cum* 'together with' and *crēscere* 'to grow', pointing out that creativity is the principle of *novelty*.[171] This creative growing together produces forms and structures that are quite new, that have never been seen before.

In contrast, Rupert Sheldrake points out in *The Presence of the Past* that once a particular behaviour pattern is formed in evolution, it tends to repeat itself through habit. Habits are not easy to break once they are formed through what he called morphic resonance acting within morphogenetic fields, called structure-forming relationships in the Unified Relationships Theory.

We can see all these patterns in noogenesis as well as biogenesis. Most significantly, the rate of evolutionary change in society today is accelerating exponentially, because it is being driven by the creative energies of scientists and technologists, mostly unchecked by physical constraints. We can see this most clearly in the way that the number of apps, hosts, and interconnections in the Internet has been increasing at hyperexponential speeds during the past few decades. Another example is Moore's Law, named after Gordon E. Moore, the co-founder of Intel, the chip manufacturer, who published a paper in 1965 indicating that computers would double in power every two years.[172]

However, what is happening to humanity right now is not new, as David Attenborough explained in *Life on Earth*, broadcast by the BBC in 1979. It is now some 3.6 billion years since the first self-reproducing forms of life appeared on this planet. So if we consider 10 million years to be a day, we can map the whole of evolution on this planet to the days of the year.[173]

Using this model, if 1st January marks the birth of single-cell organisms, then the first multicellular organisms appeared in the middle of August, with sexual reproduction beginning about six weeks later. Other significant events during the late autumn were the emergence of fish, land plants, and reptiles. Then about the 10th December, both mammals and dinosaurs appeared, with mammals surviving the mass extinction that occurred on Christmas Day, one of seven and nine mass extinctions of land and marine forms of life so far in the life of the Earth according to the *Times Atlas*.[174]

This catastrophe enabled the primates to appear on Boxing Day, to be followed by the hominids four days later. Then on New Year's Eve, the first exemplars of the *Homo* genus appeared around teatime. The whole of human evolution has thus taken place during the evening of the last day of the year, with *Homo sapiens* being born about 23:59:30. As we rapidly approach midnight on 31st December, we can see that the whole of mental evolution in the noosphere has taken place during the last three or four seconds, with the computer age beginning less than a single tick of the clock earlier.

Peter Russell provides a similar metaphor in *The White Hole in Time*[175] and its sequel *Waking up in Time*. He uses the 108 floors of the 400-metre-high former World Trade Center in New York as a measuring stick for evolution since the formation of the Earth some 4.6 billion years ago.[176] In *The Awakening Earth*[177] and its sequel *The Global Brain Awakens*, Peter extends his view of evolution still further back. To get a complete picture, we need to look at evolution as starting from the most recent big bang, some fourteen billion years ago.[178]

This means that we need a much more generalized view of evolutionary processes than that through which biologists have viewed evolution since the publication of Darwin's mistitled *Origin of Species* in 1859. To view our lives within the overall scheme of things, we need to stand outside ourselves, viewing the Totality of Existence from a Holoramic 'Whole-seeing' vantage point. A good starting point for this holistic perspective is Pierre Teilhard de Chardin's four-stage model of evolution, brought up to date in my book *The Four Spheres*. In summary, here is Teilhard's model, as presented in that book.

Evolutionary stages, years ago						Transition stages, years ago		
Teilhard	**Type**	**Realm**	**Start**	**End**	**Duration**	**Start**	**End**	**Duration**
Prelife	Physical	Hylosphere	14,000,000,000	4,500,000,000	9,500,000,000	4,500,000,000	3,500,000,000	1,000,000,000
Life	Biological	Biosphere	3,500,000,000	25,000	3,500,000,000	25,000	5,000	20,000
Thought	Mental	Noosphere	5,000	50	5,000	50	-50	100
Superlife	Spiritual	Numinosphere	-50	-300	250			

The key point about this model is that humanity is currently in a transition process between the third and fourth stages of evolution, which Teilhard prophesied would happen when all the divergent streams of evolution converge in a megasynthesis of everything. We would then experience and see a great awakening of intelligence, as evolution becomes increasingly conscious of itself in the manner that Julian Huxley foresaw in 1957 in a short essay titled *Transhumanism*.[179] Thus, evolution is destined to evolve into superconsciousness through Teilhard's law of complexity-consciousness: the greater the complexity, the greater the consciousness,[180] complexity theory being the dual of chaos theory in some sense.

Now, Teilhard also pointed out that evolution is destined to evolve into an Omega Point at the end of time, the Great Attractor of the whole of evolution, corresponding to the accumulation point in chaos theory. In recent years, some scientists have called evolution's Accumulation Point a technological singularity, as they believe machines with artificial superintelligence will largely replace humans in the

workplace. But while computers might become cleverer and cleverer, we humans can see that this technological singularity is actually a Spiritual Singularity within our species, reached as evolution passes through the most momentous turning point in its fourteen billion-year history.

During the past two or three decades, several attempts have been made to express this evolutionary model in clear mathematical terms, some of which we look at in this section. For while Teilhard's model shows that each major stage of evolution is getting shorter and shorter because of the way that evolutionary processes accelerate exponentially, it falls short of expressing this in the rigorous language demanded by modern science.

We begin this section, not with rigorous mathematics, but with a description of how the Mayan calendar has been used by evolutionaries to denote evolution's Singularity in time. For the Mayan calendar is uniquely exponential in character, with an end point that marks evolution's Accumulation Point, as the third section explains. But before this, we also look at a fractal mathematical model of the Singularity, which Terence McKenna developed called Timewave zero. Fourthly, we look at how Ray Kurzweil and others view the Singularity in time without using the rigorous mathematics described in this essay.

The Mayan calendar

The most popular understanding of the singularity that evolution is currently passing through derives from the Mayan Long Count calendar, which lasts 1,872,000 days or about 5,125 years, corresponding to the start of the patriarchal epoch, in a similar manner to several other calendars. Exactly when the Long Count is considered to have ended has been the subject of some debate, dependent on how this Mayan calendar is mapped to the Gregorian one and on what is meant by the end of the calendar.

For instance, John Major Jenkins has said, "The 'zero' date is written 0.0.0.0.0, and the 'end-date' of the 13-baktun cycle is thus written 13.0.0.0.0."[181] Similarly, Carl Johan Calleman of Dalarna University in Sweden has said that this Great Cycle begins and ends at 4 Ahau in the tzolkin calendar.[182] But these statements are like saying that the beginning and end of a year is 1st January and that the twentieth century began in 1900 and ended in 2000, when this year was actually the first in the twenty-first century, by popular decree. So what does all this mean? How can we make sense of what appears to be quite a confusing situation?

Well, the Mayans did not have just one calendar, they had three: the civic calendar was called the haab, the sacred calendar the tzolkin, and what might be called the cosmic calendar, of which the Long Count was the final part. The basic unit in all three calendars was the day, called a kin. The haab calendar consisted of eighteen 20-day periods called uinals, plus five 'waiting' days called vayeb, totalling 365 days,[183] approximately matching the solar year. The uinals acted like months, shorter than the usual lunar months, the first three days in the cycle being 1 Pop, 2 Pop, and 3 Pop.[184]

The tzolkin calendar was rather strange. A day in this calendar was denoted by a number and a glyph, there being 13 numbers and 20 glyphs giving 260 days in all. But these were not ordered like days in a month, as in most calendars. The first three days were 1 Imix, 2 Ik, and 3 Akbal, as if the first three days of the Gregorian calendar were 1 January, 2 February, and 3 Mars. For instance, the 160th day was 4 Ahau.

Apparently, neither of these calendars had a way of counting years. For many purposes they were combined into a cycle of 18,980 days, the least common multiple of 260 and 365, nearly 52 years, called the calendar round or short count. A typical calendar round date was 8 Cauac 2 Zip, denoting days in the tzolkin and haab calendars, respectively.

The Long Count, also called the 'Great Cycle' or 'World Age', is 13 baktuns in length, each baktun being divided into 20 katuns, 20 tuns, 18 uinals and 20 kins. So a ninth-century date, when the Mayans were in their heyday, could be 9.18.2.5.17, like saying that 17th June 2003 is 2003.6.17. The first day in the Long Count is thus 0.0.0.0.0 and the last day is 19.19.19.17.19, corresponding to the last day of the twentieth century, for instance: 1999.12.31, 31st December 1999.

As we can see, the Long Count is almost exponential, the one exception being the 18 uinals in a tun, to make a 'year' of 360 kins, rather than 400. However, as such, it does not enable us to map Teilhard's four-stage model of evolution as whole. To do this, we need to look at the Long Count in the context of the cosmic calendar, as Carl Johan Calleman has done. The Mayans also had longer periods of time in their cosmic calendar, called piktuns, kalabtuns, kinchiltuns, alautuns, and hablatuns, each twenty times longer than the later one. For instance, a piktun is 20 baktuns, much longer than the 13 baktuns in the Great Cycle.

But the Mayans did not stop there in their vast view of time. A stele has been found at Cobá in the Yucatán peninsula that gives the date of Creation as 13×20^{21} tuns ago,[185] which is about 27 octillion years (27 followed by 27 zeroes), double the order of magnitude of the Hindu view of age of the Universe as 100 Brahma years, which is 311 trillion years, or 14 orders of magnitude, as we see on page 23. Carl Johan has denoted the Mayan cosmic view in this symbol:

13.13.13.13.13.13.13.13.13.13.13.13.13.13.13.13.13.13.13.13.13.0.0.0.0[186]

Although the Mayan cosmic calendar is vigesimal, based on 20, rather than the 10 of our decimal system, it seems that this stele measured cycles of each period of time, perhaps because there were thirteen gods in the Mayan pantheon. This explains why there are only thirteen baktuns in the Great Cycle and not twenty. However, 13 does not appear in the Long Count except as a measure of baktuns at the highest level. To get round this problem, Carl Johan looks at the Great Cycle as 13 cycles of baktuns, katuns, tuns, 18-day periods, and kins. These 18-day periods, two days shorter than uinals, don't have a name in any Mayan calendar, but are needed to make the exponential model fit.

Now Carl Johan could map the starting dates of the nine major creation cycles in the Mayan calendar to significant evolutionary turning points since the most recent big bang, as this table shows, omitting the last thirteen kins before the singularity. [187]

Underworld	Cycle	Formula	Years before singularity	Initiating phenomenon	Modern dating in years
Cellular	hablatun	13×20^7 *tun*	16,401,171,606	First matter, 'Big Bang'	13.7 billion
Mammalian	alautun	13×20^6 *tun*	820,058,580	First animals	850 million
Familial	kinchiltun	13×20^5 *tun*	41,002,929	First monkeys	40 million
Tribal	kalabtun	13×20^4 *tun*	2,050,146	First humans	2 million
Regional	piktun	13×20^3 *tun*	102,507	Spoken language	100,000
National	baktun	13×20^2 *tun*	5,125	Written language	5,100
Planetary	katun	13×20^1 *tun*	256	Industrialism	(1769)
Galactic	tun	13×20^0 *tun*	13	?	?
Universal	18-kin	13×18 *kin*	1	?	?

These nine underworlds apparently correspond to the nine levels in the Pyramid of Kukulcan in Chichen-Itza, on the Yucatán peninsula, depicted on the next page at the spring equinox. For at this time of the year, it is possible to watch the descent of the Plumed Serpent, in a cycle of seven light waves and six dark ones. Carl Johan calls these seven days and six nights in the cycles in each underworld, extensively mapping these thirteen cycles to significant points in evolutionary history, especially the fifth day and

night, as Ian Lungold shows on YouTube[188] and Keith Wyatt shows in his video 'The Quickening' on his website Awakening as One.[189]

Kukulcan is the Mayan name for what the Aztecs called Quetzalcoatl 'feathered serpent', in whose name prophecies have been made for the dawn of an eschatological Golden Age at the end of the Mayan calendar, not unlike the millennial prophecies of some Christians and other religionists. But when is this going to happen? Well, it is not possible to calculate this using the formula for convergent geometric series on page 5, for there is no reference point to our Gregorian calendar. This is because by the time that the Spaniards conquered Mesoamerica, the Mayans had abandoned the Long Count calendar.

So what Mayan scholars do is use Julian day numbers, which astronomers use to make predictions such as solar and lunar eclipses. Astronomers regard the zero point of their numbering system to be 12:00 UT on Monday 1st January 4713 BCE in the proleptic Julian calendar (*proleptic* means that it is applied to cases from before it was invented), or 24th November 4714 BCE in the proleptic Gregorian calendar. This is taken as the beginning of recorded history. Using this way of measuring time, Saturday, 1st January 2000 had a Julian day number of 2,451,545, which we can call the correlation coefficient when matching the Mayan calendar to the Gregorian calendar.

In Mayan terms, many long-count dates have been discovered with the form 9.w.x.y.z, denoting contemporary events in the fifth to ninth centuries. So we know roughly when the tenth baktun existed and so can work backwards to the beginning of the Long Count. But can we be more accurate? Can we find the precise Julian day or correlation coefficient that marks the beginning of the Great Cycle of 13 baktuns, 5,125 years, or 1,872,000 days?

Well, after many years of considering information from various fields such as astronomy, ethnography, archaeology, and iconography, J. Eric S. Thompson found a correlation coefficient of 584,283, which is now known as the Goodman-Martinez-Thompson (GMT) correlation.[190] This gives the first and last days of the Long Count as Monday 11th August 3114 BCE and Thursday 20th December 2012, respectively. The next day, the winter solstice, is the first day *after* the end of the calendar, when a New Age is supposed to dawn. However, Floyd Lounsbury, supported by Linda Schele, David Freidel, and a number of others, promoted 584,285 as the correlation coefficient, giving 23rd December 2012 as the first day after the end of the calendar.[191] I don't know the reasons for this.

To reconcile these two ways of calculating the end of the Mayan calendar and the birth of a quite new way of living our lives, as evolution becomes increasingly conscious of itself, evolutionaries like Barbara Marx Hubbard used the winter solstice of 2012 as the last day of the Mayan calendar in her Birth2012 initiative, attracting a great following from around the world. Together with Stephen Dinan, CEO of the Shift Network, and many others, this has now evolved into Vision2020, indicating that we have just a few years left to prepare for the immense changes that are likely to happen in the 2030s and 2040s.

Timewave zero

Let us now look at another way of showing how all evolutionary processes in the Universe are leading to a singularity of time, a model that I discovered from Peter Russell[192] in 2011 and which has gained popularity on the Web. In 1971, 24-year-old Terence McKenna and his 20-year-old brother Dennis travelled to Amazonian Columbia to study ethnobotany, the way that various plants are used by shamans

to induce psychedelic transformations in consciousness, *psychedelic* deriving from Greek *psyche* 'soul, mind' and *dēlos* 'clear, visible', from PIE base **dyeu-* 'to shine', also root of *divine* and *jovial*.

The McKennas were drawn to the Amazon because they had read a report that said, "shamans, under the influence of potent monoamine oxidase-inhibiting, harmine- and tryptamine-containing *Banisteriopsis* infusions, are said to produce a fluorescent violet substance by means of which they accomplish their magic." Dennis, in particular, who was later to receive a doctorate in psycho-pharmacology, speculated that such substances could transform genetic archetypes through changes in the waveform hologrammatic configuration of ESR (electron spin resonance). Such a macro-molecule "would be a superconductive holographic information storage system, containing all genetically and experientially coded information within its waveform pattern. It would respond to thought, which would be an interference pattern set up by resonating tryptamine-RNA complexes."

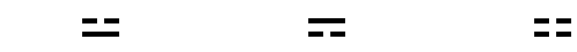
To test his hypothesis, on 4th March 1971, at a tiny mission settlement at La Chorrera, Dennis and Terence embarked on an experiment, which turned out to be life-changing. They ingested some mushrooms (*Stropharia cubensis*), whose major psychoactive constituent is psilocybin, and drank a beverage of *ayahuasca*, from the leafy, woody plant *Banisteriopsis caapi* containing harmine and tryptamine.[193] The effect was mind-shattering, which you can read about in Terence's book *True Hallucinations*, from 1993, and the brothers' book *The Invisible Landscape*, first published in 1975, but republished in 1994 with more mathematical information about the singularity in time that was revealed to Terence, in particular.

In essence, it seems that Terence, who Jay Stevens describes as a 'quicksilver poet-philosopher', almost immediately opened up to the entire Cosmos, seeing time as a series of hierarchical timewaves, resonating with each other within greater and lesser timespans, somewhat like fractals, with their property of self-similarity. Within a month of this life-changing experience, as he returned to Berkeley, Terence came "to realize that the internal logic of the timewaves strongly implied a termination of normal time and an end to ordinary history".[194]

But how could Terence make sense of this vision? He and Dennis had been educated in the USA, within the delusional worldview of Western civilization. However, they were also well aware of the great movement towards a fundamental paradigm shift in science, one that embraces Eastern mysticism and ancient wisdom, going far further than the scientific revolution of the sixteenth and seventeenth centuries. Following this emerging zeitgeist, Terence turned to *I Ching* '*Book of Changes*', in which to describe his vision.

Richard Wilhelm, the interpreter of a classical translation of the *Book of Changes*, describes *I Ching* as a collection of linear signs used as oracles, which traditionally confined themselves to answers 'yes' and 'no'. *I Ching* began in this way, with an unbroken line (–) denoting 'Yes' and a broken line (--) 'No'.[195] These lines also represent *yin-yang*, with yang being unbroken, depicted in traditional and simplified Chinese with these signs, respectively: 陰陽 and 阴阳. Yin and yang are often associated with female and male and with dark and light, respectively. But these opposites were far more general, corresponding to *A* and *~A*, respectively, in the Principle of Unity.

However, as Wilhelm tells us, "the need for greater differentiation seems to have been felt at an early date, and the single lines were combined in pairs," the lower line being more significant:



Greater yang Lesser yang Lesser yin Greater yin

Seeking even greater differentiation, a third line was added to form trigrams, which "were conceived as images of all that happens in heaven and on earth. … The eight trigrams are symbols standing for transitional states … [they] therefore are not representations of things as such but of their tendencies in movement." The trigrams were given various names and characteristics, as in this table.

Sign	Name				Characteristic	Image	Family relationship
☰	Ch'ien	quán	乾	the Creative	strong	heaven	father
☷	K'un	kūn	坤	the Receptive	devoted, yielding	earth	mother
☳	Chên	zhèn	震	the Arousing	inciting movement	thunder	first son
☵	K'an	kǎn	坎	the Abysmal	dangerous	water	second son
☶	Kên	gèn	艮	Keeping still	resting	mountain	third son
☴	Sun	xùn	巽	the Gentle	penetrating	wind, wood	first daughter
☲	Li	li	離	the Clinging	light-giving	fire	second daughter
☱	Tui	duì	兌	the Joyous	joyful	lake	third daughter

However, the Chinese did not stop there. To achieve still greater multiplicity, they combined the trigrams to form 64 (2^6) hexagrams. As Hellmut Wilhelm, Richard's son, tells us, "The system of existence and events underlying the *Book of Changes* lays claim to completeness. The book attempts a correlation of situations in life in all strata, personal and collective, and in all dimensions. An added feature of the system are the trends of development latent within the various situations and their reciprocal relations."[196]

It was this sense of wholeness that drew Terence McKenna to *I Ching* in which to express his psychedelic vision. As he said, "The *I Ching* is a mathematical divinatory tool of great age whose probable origin is the mountainous heart of Asia—the home of classical shamanism and Taoist magic". So as "divination is the especial prerogative of the shaman, whatever the cultural context … the unconscious contents which our experiment made accessible were constellated around the *I Ching* because it is particularly concerned with the dynamic relationships and transformations that archetypes undergo."[197]

But what sequence of hexagrams should McKenna use for his fractal view of time, terminating at a singularity? As there are 64 different hexagrams, there are 64! different ways of arranging them in a sequence, which is about 1.27×10^{89} or exactly:

126,886,932,185,884,164,103,433,389,335,161,480,802,865,516,174,545,192,198,801,894,375,214,704,230,400,000,000,000,000

Perhaps the most obvious way of ordering the hexagrams in this digital age is from 0 to 63 (000000 to 111111 in binary notation), 0 being yang, the unbroken line. This is the sequence that Shao Yung studied in the eleventh century, during the Sung (Song) dynasty. Shao Yung is regarded as the founder of the idealistic school,[198] focused much more on iconographic and cosmological concepts than on traditional literalistic and moralistic concepts, followed by his contemporaries.[199] It was this sequence, laid out in an 8 x 8 table, that Leibniz studied, establishing amazing parallelisms between Eastern and Western thought.[200]

However, this arrangement overlooks the reciprocal nature of the hexagrams, which can be arranged in pairs in two ways: (1) in complementary pairs, like ䷀ and ䷁ and (2) in inverted pairs, like ䷂ and ䷃. In the first of these, all six lines change in every pair, whereas in this example of inverted pairs, only two lines change. There is thus greater variety in the second arrangement and therefore more information. However, this arrangement does not work in eight cases, when the hexagram is palindromic, the same when inverted, like ䷓, when its complement is used: ䷒. Nevertheless, there are still $2^{32} \times 32!$ possible arrangements of these inverted pairs, or 1.13×10^{45}, about one quintillion cubed, a quindecillion:

1,130,138,339,199,322,632,554,990,773,529,330,319,360,000,000

So around 1000 BCE, when the *I Ching* came into wide use, the Chinese had many arrangements of the pairs to choose from. The oldest of these is known as the King Wen sequence, listed on page 56, also the series of transitions that Richard Wilhelm presented in his translation of *I Ching*. But why this sequence? What is special about it? How could this particular sequence of universal categories or archetypes shed light on one's fate? Well, McKenna discovered three interesting properties:

1. There are no transitions with a value 5.
2. A transition value of 1 is only used when the alternative would violate rule 1.
3. There is a ratio of three to one in the even and odd transitions.[201]

Nevertheless, McKenna discovered that these properties are very far from random. He generated 1.2 million random inverted pairs on a computer and found that only 805 had these three properties, 0.07%, or 1 in 1,769 Wen-like sequences. So he was quite content to use the King Wen sequence of transitions for his studies into novelty theory in fractal time, listed in this table of first-order differences.[202]

6	2	4	4	4	3	2	4
2	4	6	2	2	4	2	2
6	3	4	3	2	2	2	3
4	2	6	2	6	3	2	3
4	4	4	2	4	6	4	3
2	4	2	3	4	3	2	3
4	4	4	1	6	2	2	3
4	3	2	1	6	3	6	3

McKenna also noticed that not only are there 64 hexagrams in the *I Ching*, there are also 384 (6 × 64) lines. Now, according to Joseph Needham, from an examination of oracle bones dating to the thirteenth century BCE, the Chinese knew that the length of a lunation is 29.53 days, compared to 29.530588, as is known today. So the ancients knew that thirteen of these lunations are 383.89 days (13 × 29.53), a pretty accurate correspondence on which to base a calendar.[203]

Knowing the Chinese love of cycles, hierarchies, and resonances, McKenna then surmised that what can be done with the *yao* (lines) could also be done with the entire set of *yao*. So he hypothesized a set of resonances based on 384 × 64 days, 384 × 64 × 64 days, and so on. Not only this, he saw time in shorter and shorter durations as well as longer and longer ones. So using 6 days as the base, he found 26 levels and durations of temporal hierarchy of the form 6×64^i days, where i ranges from 7 to -18 or of the order of 10^{18} to 10^{-27} when measured in seconds. In physical temporal terms, these range from five times longer than the time since the most recent big bang to the range of Planck's constant.[204]

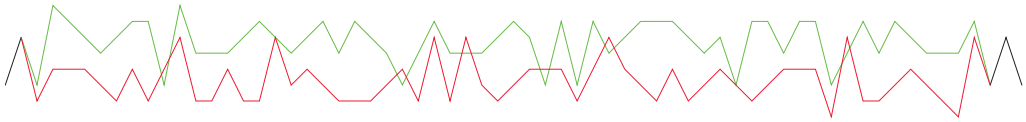
With these premises, McKenna now needed to express his resonating novelty theory in mathematical terms. In essence, he saw time "as the ebb and flow of two opposed qualities; novelty and habit, or density of connectedness versus disorder". So even though he believed in the absolute truth of the second law of thermodynamics, he saw that in localized areas entropy could decrease through *concrescence* 'growing together', a recent instance being the appearance of language.[205]

However, it was not until 1986 that McKenna began working with Peter Meyer to develop software that could translate the former's mathematical intuitions into C and thus define the core algorithm in Timewave theory. So, even though McKenna had other programming assistants, listed in full in Meyer's

1 Ch'ien	2 K'un	3 Chun	4 Mêng	5 Hsü	6 Sung	7 Shih	8 Pi
9 Hsiao Ch'u	10 Lü	11 T'ai	12 P'i	13 T'ung Jên	14 Ta Yu	15 Ch'ien	16 Yü
17 Sui	18 Ku	19 Lin	20 Kuan	21 Shih Ho	22 Pi	23 Po	24 Fu
25 Wu Wang	26 Ta Ch'u	27 I	28 Ta Kuo	29 K'an	30 Li	31 Hsien	32 Hêng
33 Tun	34 Ta Chuang	35 Chin	36 Ming I	37 Chia Jên	38 K'uei	39 Chien	40 Hsieh
41 Sun	42 I	43 Kuai	44 Kou	45 Ts'ui	46 Shêng	47 K'un	48 Ching
49 Ko	50 Ting	51 Chên	52 Kên	53 Chien	54 Kuei Mei	55 Fêng	56 Lü
57 Sun	58 Tui	59 Huan	60 Chieh	61 Chung Fu	62 Hsiao Kuo	63 Chi Chi	64 Wei Chi

documentation, it was not until then that McKenna's rather obscure vision of resonant timewaves could be expressed in a fractal function.[206]

Nevertheless, the first step is quite simple. McKenna drew a graph of the 64 hexagrammatic transitions or first-order differences in the table on page 55, shown below in red. He then rotated this graph 180° and cycled it by one position, shown in green, so that three lines matched at the ends. He called these three levels of closure, marked in black, the key to calculating his way of viewing a singularity in time.



As McKenna considered that trigrams and hexagrams should be treated in exactly the same way as lines, he next expanded this pair of waves, shown on the next page. He called this the 'eschaton', from Greek *eskhatos* 'last', also the root of *eschatology*. However, he defined *eschaton* as 'a universal and fractal morphogenetic field', a quantized wave-particle of time.[207] The eschaton is formed by linearly arranging six versions of the basic wave, two versions of the wave expanded three times, representing trigrams, and one version expanded six times, representing the hexagram as whole.

With these premises, McKenna now needed to express his resonating novelty theory in mathematical terms. In essence, he saw time "as the ebb and flow of two opposed qualities; novelty and habit, or density of connectedness versus disorder". So even though he believed in the absolute truth of the second law of thermodynamics, he saw that in localized areas entropy could decrease through *concrescence* 'growing together', a recent instance being the appearance of language.[208]

He next found the differences between the distances and slopes of each of the three pairs of waves, the slopes being the second order of differences between the transitions, which could be either positive or negative depending on the direction of the skew. The documentation on Peter Meyer's C program that performs these calculations doesn't make the underlying semantics crystal clear. So I've written a Python program to illustrate what is a rather inelegant algorithm, listed on the next page.

Sometime before Meyer's program became available, Matthew Watkins discovered McKenna's transformation of the hexagrammatic transitions and developed a single formula for what he called a 'piecewise linear function', which he described as worryingly arbitrary and mathematically clumsy, lacking a sound foundation. He was particularly critical of the sign reversal in the first 32 slope differences, known as the 'mysterious half twist', which he said invalidated the entire enterprise.[209]

However, we need to remember that McKenna was not trying to prove anything mathematically, following Euclid's linear method of proof, based on assumed axioms. Rather, he was expressing in mathematical terms the harmony of the Universe that had been revealed to him during his psychedelic trip, rather like the way that Mozart composed symphonies. Timewave Zero is a divine revelation, just as this essay is.

All the differences in the distances and slopes between the wave function and its inverse are then totalled to produce 384 data points for the timewave fractal transform:

0	0	0	2	7	4	3	2	6	8	13	5	26	25	24	15	13	16	14	19	17	24	20	25
63	60	56	55	47	53	36	38	39	43	39	35	22	24	22	21	29	30	27	26	26	21	23	19
57	62	61	55	57	57	35	50	40	29	28	26	50	51	52	61	60	60	42	42	43	43	42	41
45	41	46	23	35	34	21	21	19	51	40	49	29	29	31	40	36	33	29	26	30	16	18	14
66	64	64	56	53	57	49	51	47	44	46	47	56	51	53	25	37	30	31	28	30	36	35	32
28	32	27	32	34	35	52	49	48	51	51	53	40	43	42	26	30	28	55	41	53	52	51	47
61	64	65	39	41	41	22	21	23	43	41	38	24	22	24	14	17	19	52	50	47	42	40	42
26	27	27	34	38	33	44	44	42	41	40	37	33	31	26	44	34	38	46	44	44	36	37	34
36	36	36	38	43	38	27	26	30	32	37	29	50	49	48	29	37	36	10	19	17	24	20	25
53	52	50	53	57	55	34	44	45	13	9	5	34	26	32	31	41	42	31	32	30	21	19	23
43	36	31	47	45	43	47	62	52	41	36	38	46	47	40	43	42	42	36	38	43	53	52	53
47	49	48	47	41	44	15	11	19	51	40	49	23	23	25	34	30	27	7	4	4	32	22	32
68	70	66	68	79	71	43	45	41	38	40	41	24	25	23	35	33	38	43	50	48	18	17	26
34	38	33	38	40	41	34	31	30	33	33	35	28	23	22	26	30	26	75	77	71	62	63	63
37	40	41	49	47	51	32	37	33	49	47	44	32	38	28	38	39	37	22	20	17	44	50	40
32	33	33	40	44	39	32	32	40	39	34	41	33	33	32	32	38	36	22	20	20	12	13	10

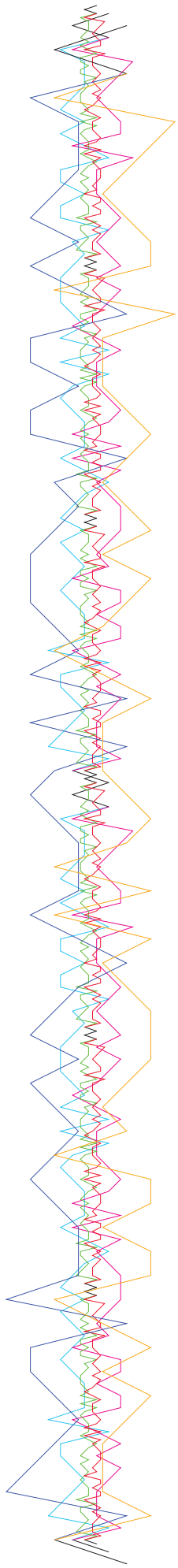
Now this list of 384 data points is both finite and discrete. So to turn it into an infinite continuous function, Meyer created a linear interpolation of these 384 values, repeated to infinity. The algorithm is given here as a Python expression, because it is clearer that way, `ds` being short for `dataSet`.

$v(x)$ = `ds[int(x)%384] + (x - int(x)) * (ds[int(x+1)%384] - ds[int(x)%384])`

Now came the master-stroke. Meyer was able to express McKenna's vision of resonant, harmonic time terminating at the end of time in a fractal transform of the function $v(x)$.[210] He first generated a general function, showing that it exists provided that two conditions are met:

1. $v(x)$ is finite for all x.
2. $v(x)$ is zero for all x less than a finite number.

The interpolated data points generated from the *I Ching* hexagrammatic King Wen transitions fit these conditions. So Meyer was able to define a specific fractal transform for Timewave Zero:



```
import copy

# list of first-order differences between hexagrams in King Wen I Ching series
diff = [6, 2, 4, 4, 4, 3, 2, 4,
        2, 4, 6, 2, 2, 4, 2, 2,
        6, 3, 4, 3, 2, 2, 2, 3,
        4, 2, 6, 2, 6, 3, 2, 3,
        4, 4, 4, 2, 4, 6, 4, 3,
        2, 4, 2, 3, 4, 3, 2, 3,
        4, 4, 4, 1, 6, 2, 2, 3,
        4, 3, 2, 1, 6, 3, 6, 3]

# rotate differences 180 degrees and offset by one position
rdiff = copy.deepcopy(diff)
rdiff.reverse()
rdiff.append(rdiff[0])
rdiff.pop(0)

# add 64th difference to start
diff65 = diff
diff65.insert(0,diff[63])
rdiff65 = rdiff
rdiff65.append(rdiff[0])

# align rotated differences with differences for closure
for i in range(65):
    rdiff65[i] = 9 - rdiff65[i]

# find the distances between the two waves
distance = []
for i in range(65):
    distance.append(rdiff65[i]-diff65[i])
distance.pop(0)

# find slopes of two waves (second order of differences)
slope=[]; rslope=[]
for i in range(64):
    slope.append(diff65[i]-diff65[i+1])
    rslope.append(rdiff65[i]-rdiff65[i+1])

# find differences between the two slopes
slopeDiff = []
for i in range(32):
    slopeDiff.append(rslope[i]-slope[i])
for i in range(32,64):
    slopeDiff.append(slope[i]-rslope[i])

# align slopes with distances
slopeDiff.append(slopeDiff[0])
slopeDiff.pop(0)


# calculate the 384 data points for the fractal transform
distLine = []; slopeLine = []
distTrigram = []; slopeTrigram = []
distHexagram = []; slopeHexagram = []
distSum = []; slopeSum = []
dataSet = []

for i in range(384):
    distLine.append(distance[i%64])
    slopeLine.append(slopeDiff[i%64])
    distTrigram.append(3*distLine[int(i/3)])
    slopeTrigram.append(3*slopeLine[int(i/3)])
    distHexagram.append(6*distLine[int(i/6)])
    slopeHexagram.append(6*slopeLine[int(i/6)])
    distSum.append(distLine[i] + distTrigram[i] + distHexagram[i])
    slopeSum.append(slopeLine[i] + slopeTrigram[i] + slopeHexagram[i])
    dataSet.append(abs(distSum[i]) + abs(slopeSum[i]))

dataSet.reverse()
```

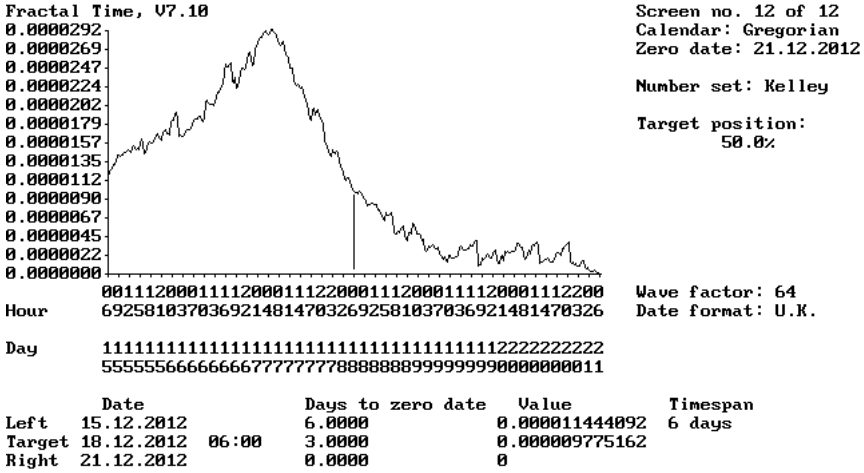
$$f(x) = \sum_{i=-\infty}^{\infty} \frac{v(x * 64^i)}{64^i}$$

He proved that this infinite series sums to a finite limit. The first condition is needed for zero and positive values of *i*. This series terminates like the formula for G_n on page 5, where $a = 79$, the maximum

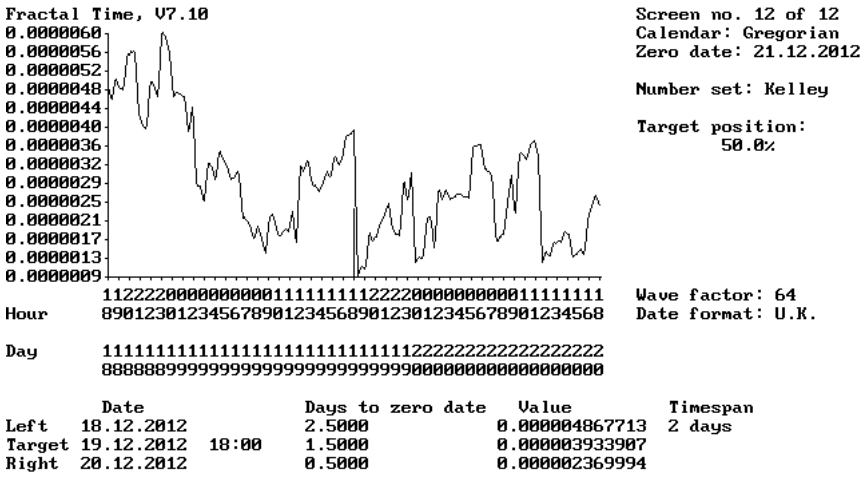
of the generated data set, and $d = 64$. For negative values of i, the second condition ensures that there are just a finite number of finite terms to be summed, for when i is absolutely greater than some finite number, the term is zero. Also $f(x) = 0$ when $x = 0$, denoting maximum novelty at zero time. $f(x)$ also has the desired resonant properties because Meyer also proved this simple relationship:

$$f\left(x * 64^{j}\right) = 64^{j} * f(x)$$

This was key. For instance, here is a plot of $f(x)$ for the six days before the zero date, amazingly generated using a 1990s DOS program under Mac OS X,[211] the days corresponding to lines in the *I Ching* hexagrams:



This wave is exactly the same as for 384 days (6 × 64), for 67 years (6 × 64 × 64), for 4306 years (6 × 64 × 64 × 64), and so on. The timewave does not need to end at the zero date. To illustrate the fractal nature of the fractal transform, we can zoom into just a part of the wave, like in a Mandelbrot set. For instance, the timewave below covers forty-eight hours from 18:00 on 18th December 2012 to 18:00 on 20th December 2012. It similarly covers 128 days from 14th July to 19th November 2012, days starting at 6:00 in the morning, and 8,192 days from 8th December 1984 to 14th May 2007.



With Meyer's *Fractal Time* software, which could now be rewritten in an interactive Jupyter notebook with Matplotlib, McKenna was able to study many periods of time, noting resonances in historical events between periods differing in length by a factor of 64. Such mappings are similar to Carl Johan Calleman's mappings to the Mayan calendar on page 51, using a factor of 20. It was these mappings that led McKenna to 2012 as the singularity in time that his vision foretold. At first, he thought that November 2012 provided the best mapping to historical events.[212] But then he discovered the projected end of the Mayan Great Count and so was happy to jump onto this bandwagon.

However, from what I have seen of the correlations between the timewave function and historical events, these are subject to much interpretation and debate. There is even much debate about McKenna's original vision for generating the data points, from which Royce Kelley and Leon Taylor developed the original algorithm in the early 1970s. For apparently, this algorithm did not exactly match McKenna's psychedelic vision. Then there is the Watkins algorithm, without the 'half-twist'. The mathematician John Sheliak has developed yet a third algorithm, quite different from the other two.[213] Furthermore, the equally ancient Huang Ti sequence of the hexagrams has a closure of 9, compared with 3 in the King Wen sequence, generating quite different timewaves.[214]

So, rather than exploring the historical resonances that McKenna studied further, it is far more relevant to note that at his singularity in time, novelty and concrescence are zero. This might seem strange, but it is easier to plot increasing novelty diminishing on a graph. There is a facility in the *Fractal Time* software to show novelty tending towards infinity, more meaningful, but not so easy to display on a finite computer display.

So what does it mean to say that novelty is now reaching its evolutionary maximum? Well, this essentially means that there are no longer any inhibitors to creativity, no paradigms or dogmatic religious, scientific, or economic worldviews preventing evolution flowing with its full power. As we see on page 62, in systems theory terms, this is like turning a tap full on so that it flows profusely at evolution's Accumulation Point. Furthermore, as concrescence also reaches a maximum at this point, all the divergent streams of evolution, which have led to our fragmented minds and schizoid society, converge in Wholeness at the beginning and end of time in the Eternal Now.

But what does it *feel* like to pass through this Singularity in time? In *The Voice of Experience*, R. D. Laing pointed out in this critique of objective science, "Experience is not an objective fact. A scientific fact need not be experienced." Furthermore, "A fact makes no difference to me personally unless I realize it. … It is very much easier to realize something one experiences personally than something one does not, perhaps cannot, experience at all."[215] So if we have never had a psychedelic experience that opens consciousness to Consciousness itself, how can we possibly know what this means? Well, when Richard Alpert, who became Ram Dass, gave a large dose of LSD to his guru in India, it had absolutely no effect.

There is thus a mystical approach to opening up to the Divine, as many spiritual teachers are showing today. One of these is Rupert Spira, a teacher of Nonduality, who says in *The Transparency of Things: Contemplating the Nature of Experience*, "If we explore Consciousness we find that it has no objective qualities. And yet it is what we most intimately know ourselves to be. … There is no adequate name for that into which the mind dissolves. We are taken to the utmost simplicity of direct experience." However, "this does not invalidate the use of the mind to explore the nature of Consciousness and Reality."[216]

However, how can we do this? For some 25,000 years, we human beings have struggled to understand the Divine, which we experience as Presence, from the Latin *præsentia*, participle of *præesse* 'to be before', from *præ* 'before' and *esse* 'to be'. The Latin origin of *presence* literally means 'before being' or 'prior to existence'. So to fully understand what it means to pass through the Singularity in time, we need both experience and a coherent, self-reflective worldview that can make sense of our experiences. Let us therefore explore this union of rationality and mysticism.

Evolution as a dynamical system

Towards the end of the 1990s, Carl Johan Calleman gave a talk on his evolutionary model to students at the Holma College of Holistic Studies in southern Sweden, where Nick Hoggard was doing a one-year

course. At first, Nick was rather sceptical of what he called a 'wild and remarkable theory'. However, when he was relaxing on a beach in Malmö on 11th July 1999, he had a eureka moment, when he realized that he could express this evolutionary model in terms of the logistic map in chaos theory, outlined in Subsection 'Robert May and chaos theory' on page 41.

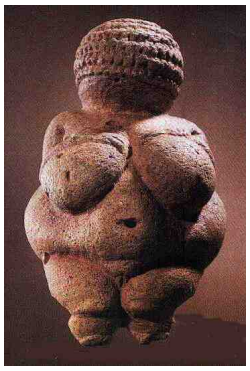
Nick did this by first noticing a glaring omission in Carl Johan's model: the so-called origin of life on Earth was not included as a major turning point. He realized that such a momentous event in evolutionary history could be included by considering $\sqrt{20}$ rather than 20 as the evolutionary diminishing factor. Now $\sqrt{20}$ is 4.472, which is reasonably close to 4.669, the mathematical constant (δ) that Mitchell J. Feigenbaum had amazingly discovered among the bifurcation points in nonlinear difference equations, in the 1970s, defined on page 46, introducing the notion of universality in dynamical systems.

Nick therefore realized that he could use systems theory to develop a comprehensive model of evolutionary history. He wasn't the first to have this idea. In *The Phenomenon of Science: A Cybernetic Approach to Human Evolution* published in 1977, Valentin Turchin, a Russian physicist, computer scientist, and Soviet dissident, used cybernetics to model the way that evolutionary structures are becoming ever more complex in their organization.[217] I first read this book in 1980, but it did not really provide me with the sound exponential model that I was seeking. Nick provided me with such a model in 2000, when I heard him give a talk in south-east Sweden at the continental meeting of the British Scientific and Medical Network.

Here is a little of what I learned from Nick, working as a software developer the last time I spoke to him a few years ago. He first converted Carl Johan's table on page 51, interpolating other significant evolutionary turning points, as shown here.

#	Event	Description	Best known date
0	Big Bang	The universe is created and matter starts to evolve into ever more complex forms. This eventually results in the organic molecules, which are needed for life to appear.	13-16 billion years ago
1	Emergence of life	Organic molecules join together in self-contained entities, which are able to replicate and mutate.	3.5-3.8 billion years ago
2	Sexual reproduction	Two organisms are able to combine their genes to produce new, novel organisms. This substantially increases the rate of biological evolution.	1,000 million years ago
3	Passing on learned behaviour (mammals)	Animals start to care for their young. This gives them an opportunity for them to pass on useful experience to the next generation. This turns out to be a faster way of evolving behaviour than waiting for behaviour to evolve purely genetically.	200 million years ago
9	Unknown event	The theory predicts a transition which has not been identified, but which perhaps explains the appearance of agriculture.	25,000 years ago
10	First civilization	Diversification of skills means that man moves on to new heights of creativity, technology, and culture.	6,000 years ago
11	First technological revolution	A wave of mechanization gathers pace across Europe and eventually the world.	500-800 AD
12	Industrial revolution	A combination of factors triggers the industrialization of society, revolutionizing daily life.	1733
13	Invention of computer	The universal machine is invented, and technological development moves from hardware to software.	1946-1948
14	Cyberspace (World Wide Web)	The invention of cyberspace connects people in an interactive environment free from the limitations of physical distance.	1991

For myself, I call the unknown event that Nick referred to the birth of Self-reflective Intelligence, the ability to look inwards into the depths of the Cosmic Psyche as well as outwards. Such a capacity marks the Divine quality that distinguishes humans from the other animals and machines, like computers. We see evidence for this momentous change in evolutionary history in the cave drawings in south-west France



and in the images of goddesses appearing across a vast expanse of land stretching from the Pyrenees to Lake Baikal in Siberia, at the time of the Great Mother Goddess. A famous example is this limestone figurine of a fertility goddess that was found in Willendorf in Austria, estimated to be between 18 and 20,000 years old.[218]

Since then, however, we have fallen far short of the immense potential that Self-reflective Intelligence gives us, most probably because of the existential fears that can arise through self-inquiry and introspection, as we see in the myth of Pandora's box. So, not knowing the joys of the mystics in their meditation practices, we have mostly interpreted evolution as the development of technology, especially during the patriarchal epoch. But today, denying the power of Self-reflective Intelligence to understand ourselves is no longer viable, given the claims that of computer scientists that they are about to develop machines with artificial general superintelligence.

We can see this most clearly when we look at these major evolutionary turning points as bifurcation points in nonlinear difference equations, as Nick did in his essay 'Superevolution'. But before we look at how this applies to evolution as a whole, it is perhaps easiest to look at an example of a complex dynamic process familiar to us all: a dripping tap or faucet, as the Americans usually say. A search of the Internet reveals several experiential and mathematical studies of this phenomenon, which displays the bifurcations of a dynamical system.

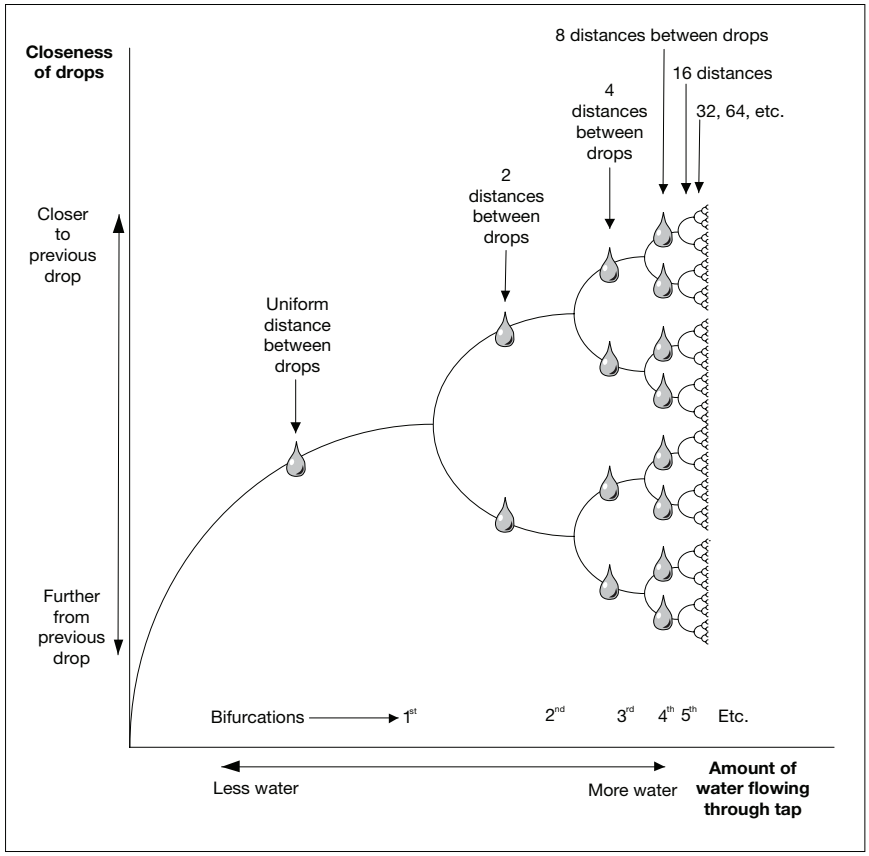
Essentially, what happens when a tap is gradually turned on under laboratory conditions is that the drips are evenly spaced. At the first bifurcation, drips fall in pairs, with a smaller distance within the pairs than between them.[219] Then at each bifurcation, the number of different distances between the drops doubles, until the tap is turned full on at its accumulation point. On the next page is an idealized schematic that Nick used to illustrate this phenomenon, with a geometric ratio of 2 rather than 4.669 to make the diagram clearer.

Now, as all bifurcating systems follow the same universal, underlying laws—from dripping taps, to population growth, to evolution as a whole—how can we calculate evolution's Accumulation Point, occurring when there are no more major evolutionary turning points to be discerned? Using the metaphor of the dripping tap, this Omega Point of evolution is reached when the tap is turned full on and no longer drips.

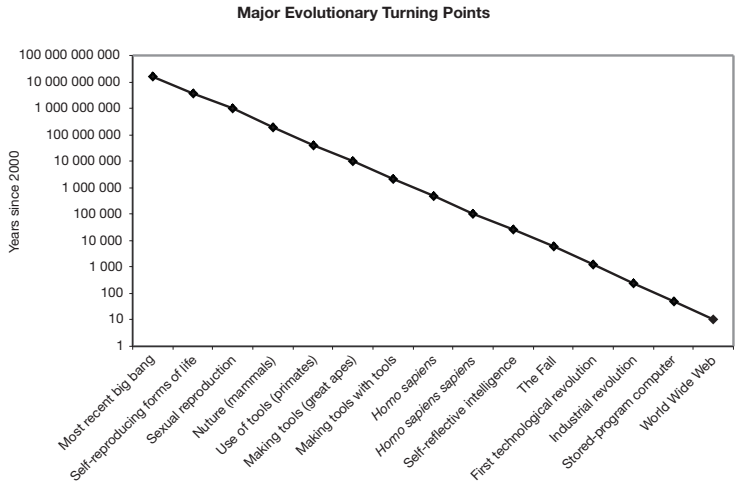
Well, the way that I have been making this calculation since 2000, when I discovered Nick Hoggard's brilliant evolutionary model, is to calculate the total length of time since the most recent big bang by this equation:

$$T = a \sum_{n=1}^{\infty} \left(\frac{1}{\delta}\right)^n$$

where δ is the Feigenbaum bifurcation velocity, defined on page 46. This formula shows that the periods between successive major evolutionary turning points are diminishing geometrically because of the accelerating exponential rate of all accumulative growth processes. However, bifurcations in nonlinear mathematics do not follow an exact geometric series, for δ is the limiting value for an infinite series of bifurcations. Nevertheless, it is near enough to use the formula for G_n on page 5. When d = 4.669 and a = 1, the finite sum is 1.273. But in evolutionary terms, a is not one, it is about ten and a half billion years, the period between the most recent big bang and the supposed birth of life on Earth.



Using a semilogarithmic scale to denote the exponential nature of accumulative evolutionary processes, Nick presented his evolutionary model in this simple diagram, which I have slightly modified.



So how can we calculate evolution's Accumulation Point from this evolutionary trend, as the infinite series of bifurcating turning points accelerates towards a finite limit? Well, we don't know the dates of most of the major evolutionary turning points in the table on page 61 with any accuracy. However, we do know the dates of the last three turning points in the above chart quite well. I have seen various dates for the start of the Industrial Revolution around the middle of the eighteenth century, so let us take a_{12} in the formula on page 46 to be 1st January 1750. We also know that the first stored-program computers ran their first programs on 21st June 1948 and 6th May 1949 at Manchester and Cambridge Universities in England, respectively. So let us take 1st January 1949 as a_{13}, giving Δ_{13} as 72,683 days. Then we know that Tim Berners-Lee put the World Wide Web online to

a_n	Date	Δ_n (days)
12	1 Jan 1750	
13	1 Jan 1949	72,683
14	6 Aug 1991	15,557
15	18 Sep 2000	3,332
16	2 Sep 2002	714
17	2 Feb 2003	153
18	6 Mar 2003	33
19	13 Mar 2003	7
20	15 Mar 2003	2
21	15 Mar 2003	0

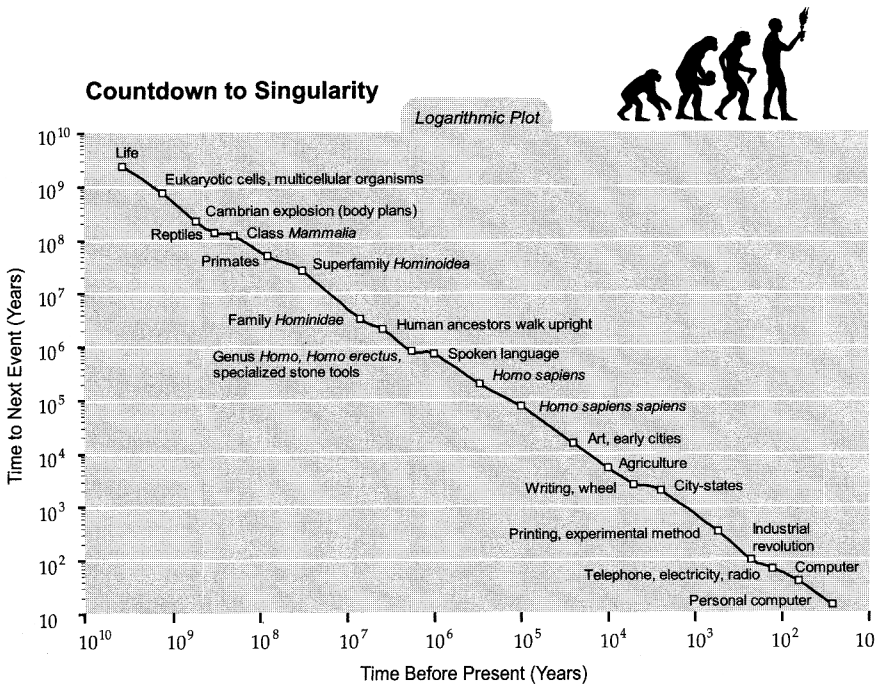
the public on 6th August 1991,[220] giving Δ_{14} as 15,557 days. So $\Delta_{13}/\Delta_{14} = 4.672$, reasonably close to δ.

As the limit of Δ_n/Δ_{n+1} is about 4.669, we can then use this figure to calculate the next few values of Δ_n, showing that Δ_{21} was equal to Δ_{20} on 15th March 2003, to the nearest day. All subsequent bifurcations took place in hours, minutes, seconds, and fractions of a second, with a_∞ happening in infinitesimal time on the same day, when the evolutionary tap is turned full on. Of course, this is a calculation of spurious accuracy. Nevertheless, using some sensitivity tests on the values of Δ_{13} and Δ_{14} still leads to evolution's Accumulation Point at the beginning of this millennium. So instead of referring to dates as BCE (before the common era) or MYA (millions of years ago), we could use the year 2000 as evolution's Accumulation Point, and refer to all dates prior to evolution's most momentous turning point as BAP (before the Accumulation Point).

I'm not sure that the major turning points from a_{15} onwards are of any special significance. These were the years when the Internet really took off, expanding at superexponential rates of acceleration. This happened because the object-oriented programming and modelling tools that application developers were using reached a reasonable level of maturity, having passed the coordination point at the bottom of the sigmoidal shape of the learning curve. We shall need to intelligently and consciously ride this wave in the coming years if we are to deal practically with all the challenges we shall face as the mainstream of society degenerates further and further into chaos, unable to adapt to our rapidly changing environment.

A technological singularity in time

Ray Kurzweil provides a similar exponential model in *The Singularity is Near*, published in 2005, presented below. The caption reads, "Countdown to Singularity: Biological evolution and human technology both show continual acceleration, indicated by the shorter time to the next event (two billion years from the origin of life to cells; fourteen years from the PC to the World Wide Web)"



Not recognizing the existence of the noosphere and numinosphere in Teilhard's evolutionary model, Ray divides evolution on Earth into six epochs of patterns of information: 'Physics and Chemistry', 'Biology and DNA', 'Brains', 'Technology', 'The Merger of Human Technology with Human

Intelligence', and 'The Universe Wakes Up', which he describes as "the ultimate destiny of the Singularity and of the universe".[221]

This is not my experience or that of those who have looked deeply inside themselves. If we are to thrive and survive for as long as possible, it is essential to open up Pandora's box to its fullest. For it is not true that technological development can drive economic growth for very much longer. But this is a message that few want to hear at the moment. The vast majority, including the politicians and business executives who govern our practical lives, prefer to follow the predictions that computer scientists are making about artificial intelligence, even though what they are saying often generates much existential fear.

For instance, in 1993, Victor Vinge made a prediction that a technological singularity would occur in just seven years from now. In a NASA paper he called 'What is the Singularity?', he said, "Within thirty years, we will have the technological means to create superhuman intelligence [in machines]. Shortly after, the human era will be ended."[222] Continuing, he said,

> From the human point of view this change will be a throwing away of all the previous rules, perhaps in the blink of an eye, an exponential runaway beyond any hope of control. Developments that before were thought might only happen in 'a million years' (if ever) will likely happen in the next century. … I think it's fair to call this event a singularity ('the Singularity' for the purposes of this paper). It is a point where our old models must be discarded and a new reality rules. As we move closer to this point, it will loom vaster and vaster over human affairs till the notion becomes a commonplace. Yet when it finally happens it may still be a great surprise and a greater unknown.[223]

Similarly, Ray Kurzweil has said, "By 2019, a $1,000 computer will match the processing power of the human brain."[224] He seems to believe that artificial intelligence is a function of the calculating capacity of computers—an inevitable consequence of the exponential nature of growth processes.

Similarly, Hans Moravec forecast in *Robot* in 1990 that robots "could replace us in every essential task and, in principle, operate our society increasingly well without us."[225] Martin Rees, former President of the Royal Society, picked up this viewpoint by writing in *Our Final Century: Will the Human Race Survive the Twenty-first Century?*, "A superintelligent machine could be the last invention that humans need ever make."[226] And again, Stephen Hawking told the BBC on 2nd December 2014, "The development of full artificial intelligence could spell the end of the human race."[227]

So when Martin Rees says, "I think the odds are no better than fifty-fifty that our present civilisation on Earth will survive to the end of the present century without a serious setback,"[228] he is grossly out in his estimation. As evolution is currently passing through its epoch-making Accumulation Point, on the edge of chaos, the chances of Western civilization surviving until even 2030 are almost zero, for no civilization in human history has been more deluded, out of touch with Reality.

It is thus of the utmost importance that we counteract the group of scientists and technologists who have founded a Singularity University, whose "mission is to educate, inspire and empower leaders to apply exponential technologies to address humanity's grand challenges."[229] To promote its belief that robotics and artificial intelligence can solve humanity's grand challenges, the Singularity University has set up a Singularity Hub with the motto 'Science, technology, the future of mankind'.[230]

But the future of humanity does not lie in the development of more and more technology. I am not a Luddite in saying this, for without IBM, Apple, and the Internet, I could not have reached evolution's Glorious Culmination. We humans are the leading edge of evolution, not computers. We have the potential to rise far above our machines as superintelligent, superconscious beings, rather than being their slaves, as so many are today, by reversing the inside-out, upside-down culture we live in today.

Practicalities

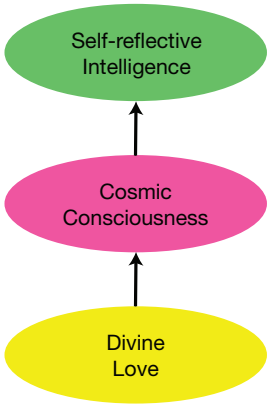
This essay on evolution's Accumulation Point completes a series of books, treatises, and essays I have written during the past five to ten years explaining what is causing scientists and technologists to drive the pace of change in society at unprecedented exponential rates of acceleration. As this is an unparalleled situation, not only in human history, but also in evolution as a whole, it requires a revolutionary practical solution.

The central problem we face as a species is that because evolution has been more divergent than convergent during the past fourteen billion years, the world we live in has become greatly fragmented and polarized, as projections of our schizoid and deluded minds. Such divisions lead to existential fear, becoming more and more obvious as evolution passes through its Accumulation Point into chaos.

The traditional way to heal our sick minds has been to kill them, leading to No-mind. Such a spiritual path is called *via negativa* in Christianity, corresponding to *jñāna-yoga*, the path of wisdom and abstract knowledge in Advaita 'not-two'. To answer the question "Who am I?", the practitioner repeats the incantation *neti neti* 'not this, not this'. However, as Ramana Maharshi pointed out, the mind cannot kill the mind.[231] By diligently studying the nature of the mind, he said that we discover that our True Nature is *Satchitānanda* 'Bliss of Absolute Truth and Consciousness'.[232]

In 2002, Paula Marvelly interviewed fifteen teachers of Nonduality, published in *The Teachers of One*. These were Satyananda, Wayne Liquorman, Pratima, Tony Parsons, Francis Lucille, Vijai Shankar, Mira, Bharat, Catherine Ingram, John de Ruiter, Pamela Wilson, Isaac Shapiro, Vartman, Ganganji, and Ramesh Balsekar. I've met a couple of these and listened to or read the words of a few others as mirrors to my own awakening. Two other teachers of Nonduality who I know quite well are Rupert Spira, a popular guide at the Science and Nonduality (SAND) conferences, and Nukunu (Jørgen Larsen), who has recently moved to the community where I live, Vartman (Alex Perrett) having previously lived with us here in western Sweden.

However, killing the mind does not enable us to deal intelligently with the practicalities of our global village, a term that Marshall McLuhan coined in the early 1960s to denote our wired society, following conversations with Wyndham Lewis, author of *America and Cosmic Man*, and James Joyce, author of *Finnegans Wake*.[233] Rather, what we need to do is heal our fragmented minds in Wholeness, with what Aurobindo called 'Supermind', using the Internet as a mirror for our minds. As Confucius said, the way to peaceful, harmonious governance is a life of learning, grounded in the *Dao*, the 'Way'.



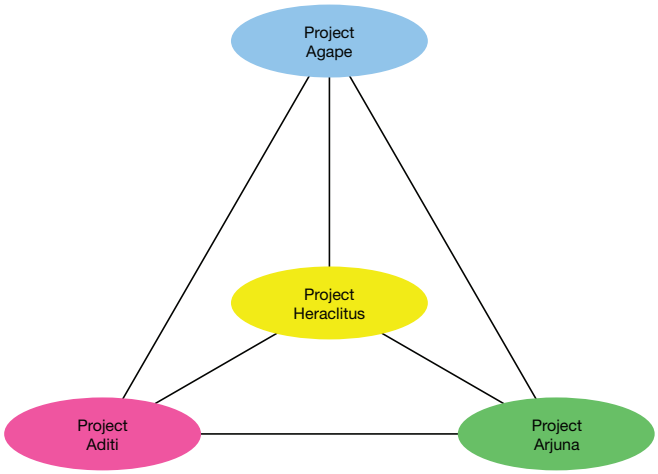
However, just as the mind cannot kill the mind, the mind cannot heal itself. Rather, only the Divine power of Life and the Logos can heal our grievously sick society, being expressed as Divine Love, Cosmic Consciousness, and Self-reflective Intelligence, acting in the vertical dimension of time, not the more usual horizontal.

Such a perspective lies at the heart of today's cultural and scientific revolution, for in Reality there is no time. Past and future are just abstractions from or appearances in Consciousness, which provides the Cosmic Context and Gnostic Foundation for all our lives. This is the key to the practicalities that we face today. As mystics through the ages have discovered, the Cosmos is designed in such a way as to enable superintelligent, superconscious beings to be fully aware of evolution within themselves.

This does not mean that there is an intelligent designer separate from humanity. As we all live in constant union with the Divine, while the Universe is intelligently designed, there is no designer thereof.

Being conscious of how the Universe is designed, it is thus up to all of us to collectively redesign society in harmony with the fundamental laws of the Universe.

That is a primary purpose of the Alliance for Mystical Pragmatics, being guided by the motto 'Harmonizing evolutionary convergence'. As mentioned in the Introduction, the Alliance intends to coordinate four great movements in the world today: Spiritual Renaissance, Scientific Revolution, Sharing Economy, and World Peace. Corresponding to these constituents, I am in the process of defining the aims of four projects, Agape, Aditi, Arjuna, and Heraclitus, the last being the one that integrates the others into a coherent whole.



However, as I am still working in almost complete solitude, it is not yet clear to me how to attract all levels of society, from the grass roots to religious, political, business, and scientific leaders to join me in this great adventure. So, for the moment, I'll post this mathematical dissertation on the Web to see what might arise in the weeks and months ahead.

As none of us in ultimate control of our destiny, as either individuals or a species, we cannot know what fate awaits us. For as Ramana Maharshi wrote to his mother as a nineteen-year-old, as 1898 turned into 1899, when she tried to persuade him to return home from Arunachala, "What is not meant to happen will not happen, however much you wish it. What is meant to happen will happen, no matter what you do to prevent it. This is certain. Therefore the best path is to remain silent."[234]

For myself, I have tried remaining silent, resting in Stillness as much as possible, but it doesn't work. Even though it is over seventy-four years since my conception, I have immense creative energy constantly pouring through me. So to maintain my health and sanity, resting in Wholeness, I need to continuously speak out, even though no one apparently can hear my words. Accordingly, in the next few weeks, I'll write an extended postscript to this essay as a separate document, outlining how Project Heraclitus and its three subprojects could bring a sense of universal order to society, currently living precariously on the edge of chaos.

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[18] Euler, *Introduction to Analysis of the Infinite*, pp. 92–100.
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